Part I

The Questions

1,001 Questions

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In this part . . .

The only way to become proficient in math is through a lot of practice. Fortunately, you have now 1,001 practice opportunities right in front of you. These questions cover a variety of calculus-related concepts and range in difficulty from easy to hard. Master these problems, and you'll be well on your way to a very solid calculus foundation.

Here are the types of problems that you can expect to see:

- Algebra review (Chapter 1)
- Trigonometry review (Chapter 2)
- Limits and continuity (Chapter 3)
- Derivative fundamentals (Chapters 4 through 7)
- Applications of derivatives (Chapter 8)
- Antiderivative basics (Chapters 9 and 10)
- Applications of antiderivatives (Chapter 11)
- Antiderivatives of other common functions and L'Hôpital's rule (Chapter 12)
- More integration techniques (Chapters 13 and 14)
- Improper integrals, the trapezoid rule, and Simpson's rule (Chapter 15)

Chapter 1

Algebra Review

Performing well in calculus is impossible without a solid algebra foundation. Many calculus problems that you encounter involve a calculus concept but then require many, many steps of algebraic simplification. Having a strong algebra background will allow you to focus on the calculus concepts and not get lost in the mechanical manipulation that's required to solve the problem.

The Problems You'll Work On

In this chapter, you see a variety of algebra problems:

- Simplifying exponents and radicals
- Finding the inverse of a function
- Understanding and transforming graphs of common functions
- Finding the domain and range of a function using a graph
- Combining and simplifying polynomial expressions

What to Watch Out For

Don't let common mistakes trip you up. Some of the following suggestions may be helpful:

- Be careful when using properties of exponents. For example, when multiplying like bases, you add the exponents, and when dividing like bases, you subtract the exponents.
- Factor thoroughly in order to simplify expressions.
- Check your solutions for equations and inequalities if you're unsure of your answer. Some solutions may be extraneous!
- It's easy to forget some algebra techniques, so don't worry if you don't remember everything! Review, review, review.

Simplifying Fractions

1–13 *Simplify the given fractions by adding, subtracting, multiplying, and/or dividing.*

1. $\frac{1}{2}+\frac{3}{4}-\frac{5}{6}$

2. $\frac{11}{3}+\frac{3}{5}+\frac{17}{20}$

3. $\left(\frac{2}{3}\right)(21)\left(\frac{11}{14}\right)$

4. $\frac{5/6}{15/22}$

5. $\frac{5}{yx}+\frac{7}{x^2}+\frac{10x}{y}$

6. $\frac{x}{x-1}-\frac{x-4}{x+1}$

7. $\left(\frac{x^2-1}{xy^2}\right)\left(\frac{y^3}{x+1}\right)$

8. $\dfrac{\frac{x^2-5x+6}{6xy^3}}{\frac{x^2+3x-10}{10x^3y^2}}$

9. $\frac{x^3y^4z^5}{y^2z^{-8}}$

10. $\frac{(x+3)^2x^4(y+5)^{14}}{x^6(x+3)(y+5)^{17}}$

11. $\left(\frac{4x^2y^{100}z^{-3}}{18x^{15}y^4z^8}\right)^0$

12. $\frac{x^4y^3z^2+x^2yz^4}{x^2yz}$

13. $\frac{(x^2y^3)^4(y^4)^0z^{-2}}{(xz^2)^3y^{-5}}$

Simplifying Radicals

14–18 *Simplify the given radicals. Assume all variables are positive.*

14. $\sqrt{50}$

15. $\frac{\sqrt{8}\sqrt{20}}{\sqrt{50}\sqrt{12}}$

16. $\sqrt{20x^4y^6z^{11}}\sqrt{5xy^2z^7}$

17. $\sqrt[3]{x^4y^8z^5}\,\sqrt[3]{x^7y^4z^{10}}$

18. $\dfrac{\sqrt[3]{8x^3y^6}\,\sqrt[5]{x^{10}y^{14}}}{\sqrt[7]{x^{14}y^{14}}}$

Writing Exponents Using Radical Notation

19–20 *Convert between exponential and radical notation.*

19. Convert $4^{1/3}x^{3/8}y^{1/4}z^{5/12}$ to radical notation. (***Note:*** The final answer can have more than one radical sign.)

20. Convert $\sqrt[3]{4x^2y}\,\sqrt[5]{z^4}$ to exponential notation.

The Horizontal Line Test

21–23 *Use the horizontal line test to identify one-to-one functions.*

21. Use the horizontal line test to determine which of the following functions is a one-to-one function and therefore has an inverse.

(A) $y = x^2 + 4x + 6$

(B) $y = |2x| - 1$

(C) $y = \dfrac{1}{x^2}$

(D) $y = 3x + 8$

(E) $y = \sqrt{25 - x^2}$

22. Use the horizontal line test to determine which of the following functions is a one-to-one function and therefore has an inverse.

(A) $y = x^2 - 4$

(B) $y = x^2 - 4, x \geq 0$

(C) $y = x^2 - 4, -2 \leq x \leq 8$

(D) $y = x^2 - 4, -12 \leq x \leq 6$

(E) $y = x^2 - 4, -5.3 \leq x \leq 0.1$

23. Use the horizontal line test to determine which of the following functions is a one-to-one function and therefore has an inverse.

(A) $y = x^4 + 3x^2 - 7$

(B) $y = 4|x| + 3$

(C) $y = \cos x$

(D) $y = \sin x$

(E) $y = \tan^{-1} x$

Find Inverses Algebraically

24–29 *Find the inverse of the one-to-one function algebraically.*

24. $f(x) = 4 - 5x$

25. $f(x) = x^2 - 4x, x \geq 2$

26. $f(x) = \sqrt{8 - 5x}$

27. $f(x) = 3x^5 + 7$

28. $f(x) = \dfrac{2-\sqrt{x}}{2+\sqrt{x}}$

29. $f(x) = \dfrac{2x-1}{x+4}$

The Domain and Range of a Function and Its Inverse

30–32 *Solve the given question related to a function and its inverse.*

30. The set of points {(0, 1), (3, 4), (5, –6)} is on the graph of $f(x)$, which is a one-to-one function. Which points belong to the graph of $f^{-1}(x)$?

31. $f(x)$ is a one-to-one function with domain [–2, 4) and range (–1, 2). What are the domain and range of $f^{-1}(x)$?

32. Suppose that $f(x)$ is a one-to-one function. What is an expression for the inverse of $g(x) = f(x + c)$?

Linear Equations

33–37 *Solve the given linear equation.*

33. $3x + 7 = 13$

34. $2(x + 1) = 3(x + 2)$

35. $-4(x + 1) - 2x = 7x + 3(x - 8)$

36. $\frac{5}{3}x + 5 = \frac{1}{3}x + 10$

37. $\sqrt{2}(x+3) = \sqrt{5}\left(x+\sqrt{20}\right)$

Quadratic Equations

38–43 *Solve the quadratic equation.*

38. Solve $x^2 - 4x - 21 = 0$.

39. Solve $x^2 + 8x - 17 = 0$ by completing the square.

40. Solve $2x^2 + 3x - 4 = 0$ by completing the square.

41. Solve $6x^2 + 5x - 4 = 0$.

42. Solve $3x^2 + 4x - 2 = 0$.

43. Solve $x^{10} + 7x^5 + 10 = 0$.

Solving Polynomial Equations by Factoring

44–47 *Solve the polynomial equation by factoring.*

44. $3x^4 + 2x^3 - 5x^2 = 0$

45. $x^8 + 12x^4 + 35 = 0$

46. $x^4 + 3x^2 - 4 = 0$

47. $x^4 - 81 = 0$

Absolute Value Equations

48–51 *Solve the given absolute value equation.*

48. $|5x - 7| = 2$

49. $|4x - 5| + 18 = 13$

50. $|x^2 - 6x| = 27$

51. $|15x - 5| = |35 - 5x|$

Solving Rational Equations

52–55 *Solve the given rational equation.*

52. $\frac{x+1}{x-4} = 0$

53. $\frac{1}{x+2} + \frac{1}{x} = 1$

54. $\frac{x+5}{x+2} = \frac{x-4}{x-10}$

55. $\frac{1}{x-2} - \frac{2}{x-3} = \frac{-1}{x^2 - 5x + 6}$

Polynomial and Rational Inequalities

56–59 *Solve the given polynomial or rational inequality.*

56. $x^2 - 4x - 32 < 0$

57. $2x^4 + 2x^3 \geq 12x^2$

58. $\dfrac{(x+1)(x-2)}{(x+3)} < 0$

59. $\dfrac{1}{x-1} + \dfrac{1}{x+1} > \dfrac{3}{4}$

Absolute Value Inequalities

60–62 *Solve the absolute value inequality.*

60. $|2x-1| < 4$

61. $|5x-7| > 2$

62. $|-3x+1| \leq 5$

Graphing Common Functions

63–77 *Solve the given question related to graphing common functions.*

63. What is the slope of the line that goes through the points (1, 2) and (5, 9)?

64. What is the equation of the line that has a slope of 4 and goes through the point (0, 5)?

65. What is the equation of the line that goes through the points (–2, 3) and (4, 8)?

66. Find the equation of the line that goes through the point (1, 5) and is parallel to the line $y = \dfrac{3}{4}x + 8$.

67. Find the equation of the line that goes through the point (3, –4) and is perpendicular to the line that goes through the points (3, –4) and (–6, 2).

68. What is the equation of the graph of $y = \sqrt{x}$ after you stretch it vertically by a factor of 2, shift the graph 3 units to the right, and then shift it 4 units upward?

69. Find the vertex form of the parabola that passes through the point (0, 2) and has a vertex at (–2, –4).

70. Find the vertex form of the parabola that passes through the point (1, 2) and has a vertex at (–1, 6).

71. A parabola has the vertex form $y = 3(x + 1)^2 + 4$. What is the vertex form of this parabola if it's shifted 6 units to the right and 2 units down?

72. What is the equation of the graph of $y = e^x$ after you compress the graph horizontally by a factor of 2, reflect it across the y-axis, and shift it down 5 units?

73. What is the equation of the graph of $y = |x|$ after you stretch the graph horizontally by a factor of 5, reflect it across the x-axis, and shift it up 3 units?

74. Find the equation of the third-degree polynomial that goes through the points (–4, 0), (–2, 0), (0, 3), and (1, 0).

75. Find the equation of the fourth-degree polynomial that goes through the point (1, 4) and has the roots –1, 2, and 3, where 3 is a repeated root.

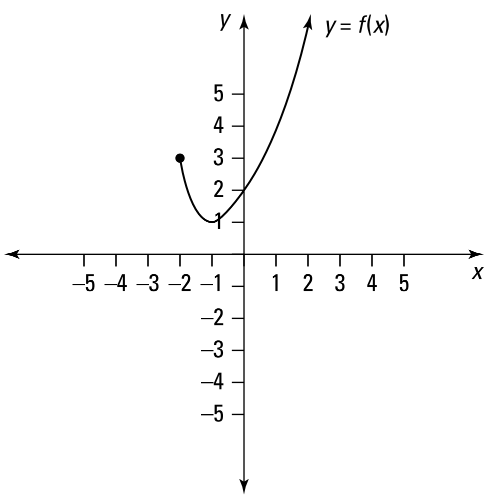
76. A parabola crosses the x-axis at the points (–4, 0) and (6, 0). If the point (0, 8) is on the parabola, what is the equation of the parabola?

77. A parabola crosses the x-axis at the points (–8, 0) and (–2, 0), and the point (–4, –12) is on the parabola. What is the equation of the parabola?

Domain and Range from a Graph

***78–80** Find the domain and range of the function with the given graph.*

78.



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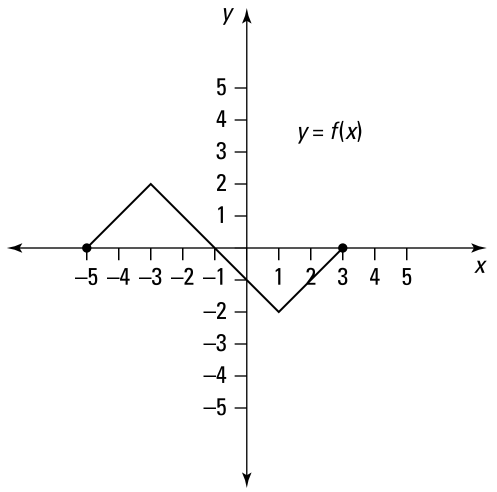

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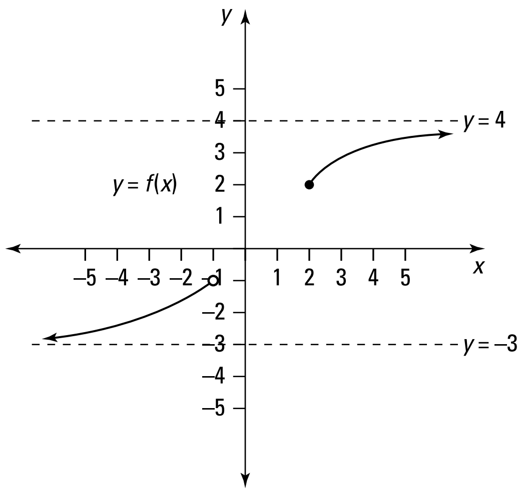


End Behavior of Polynomials

81–82 *Find the end behavior of the given polynomial. That is, find* $\lim_{x\to-\infty} f(x)$ *and* $\lim_{x\to\infty} f(x)$.

81. $f(x) = 3x^6 - 40x^5 + 33$

82. $f(x) = -7x^9 + 33x^8 - 51x^7 + 19x^4 - 1$

Adding Polynomials

83–87 *Add the given polynomials.*

83. $(5x + 6) + (-2x + 6)$

84. $(2x^2 - x + 7) + (-2x^2 + 4x - 9)$

85. $(x^3 - 5x^2 + 6) + (4x^2 + 2x + 8)$

86. $(3x + x^4 + 2) + (-3x^4 + 6)$

87. $(x^4 - 6x^2 + 3) + (5x^3 + 3x^2 - 3)$

Subtracting Polynomials

88–92 *Subtract the given polynomials.*

88. $(5x - 3) - (2x + 4)$

89. $(x^2 - 3x + 1) - (-5x^2 + 2x - 4)$

90. $(8x^3 + 5x^2 - 3x + 2) - (4x^3 + 5x - 12)$

91. $(x + 3) - (x^2 + 3x - 4) - (-3x^2 - 5x + 6)$

92. $(10x^4 - 6x^3 + x^2 + 6) - (x^3 + 10x^2 + 8x - 4)$

Multiplying Polynomials

93–97 *Multiply the given polynomials.*

93. $5x^2(x - 3)$

94. $(x + 4)(3x - 5)$

95. $(x - y + 6)(xy)$

96. $(2x - 1)(x^2 - x + 4)$

97. $-x(x^4 + 3x^2 + 2)(x + 3)$

Long Division of Polynomials

98–102 *Use polynomial long division to divide.*

98. $\dfrac{x^2+4x+6}{x-2}$

99. $\dfrac{2x^2-3x+8}{x+4}$

100. $\dfrac{x^3-2x+6}{x-3}$

101. $\dfrac{3x^5+4x^4-x^2+1}{x^2+5}$

102. $\dfrac{3x^6-2x^5-x^4+x^3+2}{x^3+2x^2+4}$

Chapter 2

Trigonometry Review

In addition to having a strong algebra background, you need a strong trigonometry skill set for calculus. You want to know the graphs of the trigonometric functions and to be able to evaluate trigonometric functions quickly. Many calculus problems require one or more trigonometric identities, so make sure you have more than a few of them memorized or at least can derive them quickly.

The Problems You'll Work On

In this chapter, you solve a variety of fundamental trigonometric problems that cover topics such as the following:

- Understanding the trigonometric functions in relation to right triangles
- Finding degree and radian measure
- Finding angles on the unit circle
- Proving identities
- Finding the amplitude, period, and phase shift of a periodic function
- Working with inverse trigonometric functions
- Solving trigonometric equations with and without using inverses

What to Watch Out For

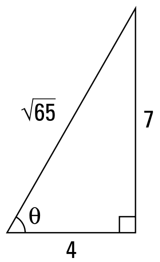
Remember the following when working on the trigonometry review questions:

- Being able to evaluate the trigonometric functions at common angles is very important since they appear often in problems. Having them memorized will be extremely useful!
- Watch out when solving equations using inverse trigonometric functions. Calculators give only a single solution to the equation, but the equation may have many more (sometimes infinitely many solutions), depending on the given interval. Thinking about solutions on the unit circle is often a good way to visualize the other solutions.
- Although you may be most familiar with using degrees to measure angles, radians are used almost exclusively in calculus, so learn to love radian measure.
- Memorizing many trigonometric identities is a good idea because they appear often in calculus problems.

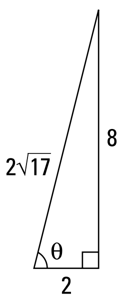
Basic Trigonometry

103–104 *Evaluate* $\sin\theta$, $\cos\theta$, *and* $\tan\theta$ *for the given right triangle. Remember to rationalize denominators that contain radicals.*

103.



104.



105–108 *Evaluate the trig function. Remember to rationalize denominators that contain radicals.*

105. Given $\sin\theta = \frac{3}{7}$, where $\frac{\pi}{2} < \theta < \pi$, find $\cot\theta$.

106. Given $\cos\theta = \frac{3}{4}$, where $\frac{3\pi}{2} < \theta < 2\pi$, find $\csc\theta$.

107. Given $\tan\theta = -\frac{8}{5}$, where $\sin\theta > 0$ and $\cos\theta < 0$, find $\sin(2\theta)$.

108. Given $\cot\theta = \frac{-9}{2}$, where $\sin\theta < 0$, find $\cos(2\theta)$.

Converting Degree Measure to Radian Measure

109–112 *Convert the given degree measure to radian measure.*

109. $135°$

110. $-280°$

111. $36°$

112. $-315°$

Converting Radian Measure to Degree Measure

113–116 *Convert the given radian measure to degree measure.*

113. $\frac{7\pi}{6}$ rad

114. $\frac{11\pi}{12}$ rad

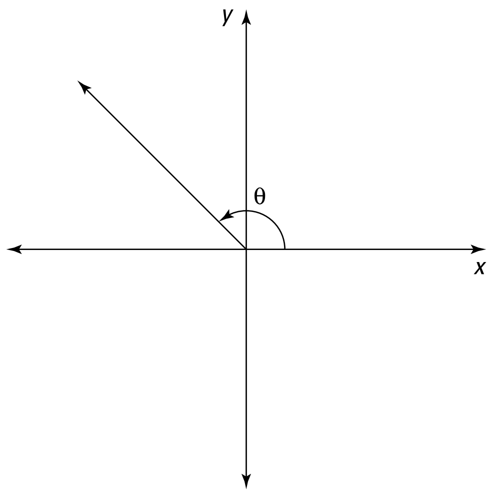
115. $\frac{-3\pi}{5}$ rad

116. $\frac{-7\pi}{2}$ rad

Finding Angles in the Coordinate Plane

117–119 *Choose the angle that most closely resembles the angle in the given diagram.*

117. Using the diagram, find the angle measure that most closely resembles the angle θ.



(A) $\frac{\pi}{3}$

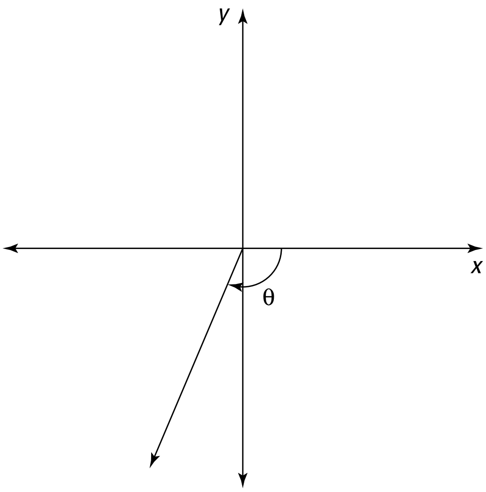
(B) $\frac{3\pi}{4}$

(C) π

(D) $\frac{7\pi}{6}$

(E) $\frac{5\pi}{3}$

118. Using the diagram, find the angle measure that most closely resembles the angle θ.



(A) $-\frac{\pi}{6}$

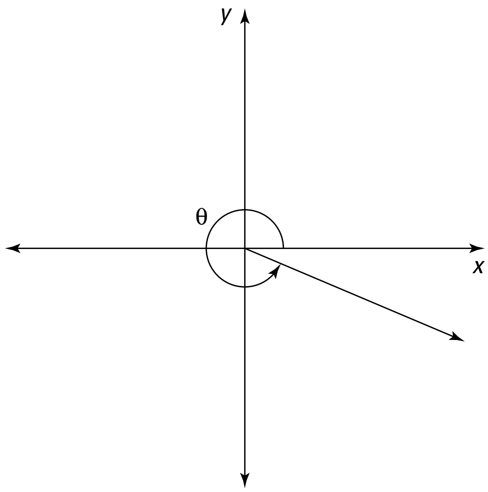
(B) $-\frac{\pi}{3}$

(C) $-\frac{2\pi}{3}$

(D) $-\frac{3\pi}{2}$

(E) $-\frac{11\pi}{6}$

119. Using the diagram, find the angle measure that most closely resembles the angle θ.



(A) $\frac{5\pi}{6}$

(B) $\frac{7\pi}{6}$

(C) $\frac{4\pi}{3}$

(D) $\frac{3\pi}{2}$

(E) $\frac{11\pi}{6}$

Finding Common Trigonometric Values

120–124 *Find* $\sin\theta$, $\cos\theta$, *and* $\tan\theta$ *for the given angle measure. Remember to rationalize denominators that contain radicals.*

120. $\theta = \frac{\pi}{4}$

121. $\theta = \frac{5\pi}{6}$

122. $\theta = \frac{-2\pi}{3}$

123. $\theta = -135°$

124. $\theta = 180°$

Simplifying Trigonometric Expressions

125–132 *Determine which expression is equivalent to the given one.*

125. $\sin\theta\cot\theta$

(A) $\cos\theta$

(B) $\sin\theta$

(C) $\sec\theta$

(D) $\csc\theta$

(E) $\tan\theta$

126. $\sec x - \cos x$

(A) 1

(B) $\sin x$

(C) $\tan x$

(D) $\cos x \cot x$

(E) $\sin x \tan x$

127. $(\sin x + \cos x)^2$

(A) $2 + \sin 2x$

(B) $2 + \cos 2x$

(C) $1 + \sec 2x$

(D) $1 + \sin 2x$

(E) $1 + \cos 2x$

128. $\sin(\pi - x)$

(A) $\cos x$

(B) $\sin x$

(C) $\csc x$

(D) $\sec x$

(E) $\tan x$

129. $\sin x \sin 2x + \cos x \cos 2x$

(A) $\cos x$

(B) $\sin x$

(C) $\csc x$

(D) $\sec x$

(E) $\tan x$

130. $\dfrac{1}{1-\cos\theta}+\dfrac{1}{1+\cos\theta}$

(A) $2\sin^2\theta$

(B) $2\tan^2\theta$

(C) $2\sec^2\theta$

(D) $2\csc^2\theta$

(E) $2\cot^2\theta$

131. $\dfrac{\sin x}{1-\cos x}$

(A) $\csc x + \cot x$

(B) $\sec x + \cot x$

(C) $\csc x - \cot x$

(D) $\sec x - \tan x$

(E) $\csc x - \tan x$

132. $\cos(3\theta)$

(A) $5\cos^3\theta - 3\cos\theta$

(B) $2\cos^3\theta - 3\cos\theta$

(C) $4\cos^3\theta - 3\cos\theta$

(D) $4\cos^3\theta + 3\cos\theta$

(E) $2\cos^3\theta + 5\cos\theta$

Solving Trigonometric Equations

133–144 *Solve the given trigonometric equations. Find all solutions in the interval [0, 2π].*

133. $2\sin x - 1 = 0$

134. $\sin x = \tan x$

135. $2\cos^2 x + \cos x - 1 = 0$

136. $|\tan x| = 1$

137. $2\sin^2 x - 5\sin x - 3 = 0$

138. $\cos x = \cot x$

139. $\sin(2x) = \frac{1}{2}$

140. $\sin 2x = \cos x$

141. $2\cos x + \sin 2x = 0$

142. $2 + \cos 2x = -3 \cos x$

143. $\tan(3x) = -1$

144. $\cos(2x) = \cot(2x)$

Amplitude, Period, Phase Shift, and Midline

145–148 *Determine the amplitude, the period, the phase shift, and the midline of the function.*

145. $f(x) = \frac{1}{2}\sin\left(x + \frac{\pi}{2}\right)$

146. $f(x) = -\frac{1}{4}\cos(\pi x - 4)$

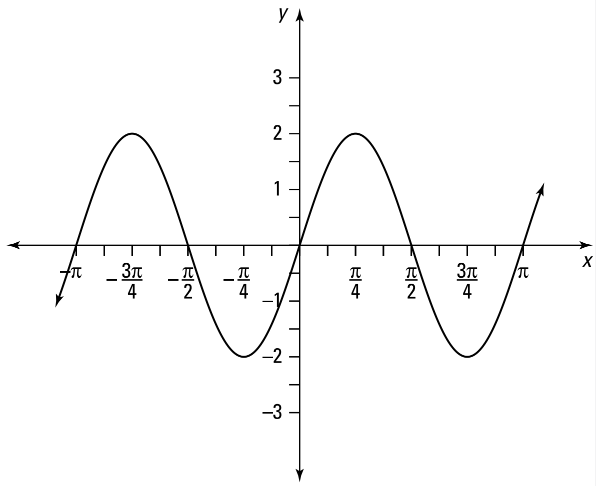
147. $f(x) = 2 - 3\cos(\pi x - 6)$

148. $f(x) = \frac{1}{2} - \sin\left(\frac{1}{2}x + \frac{\pi}{2}\right)$

Equations of Periodic Functions

149–154 *Choose the equation that describes the given periodic function.*

149.



(A) $f(x) = 2\sin(2x)$

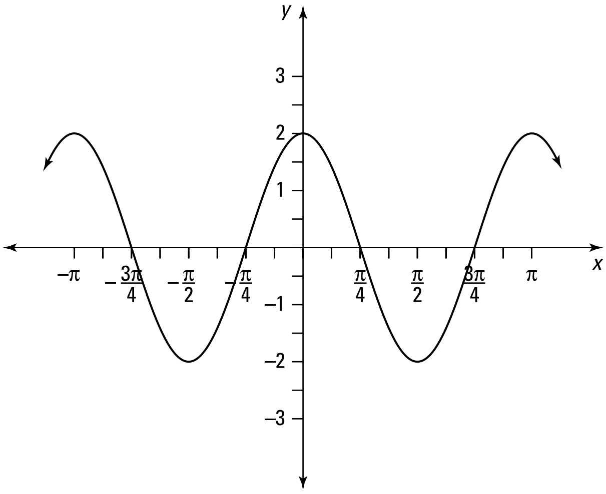
(B) $f(x) = -2\sin(2x)$

(C) $f(x) = 2\sin(x)$

(D) $f(x) = 2\sin(\pi x)$

(E) $f(x) = 2\sin\left(\frac{\pi}{2}x\right)$

150.



(A) $f(x) = 2\cos(x)$

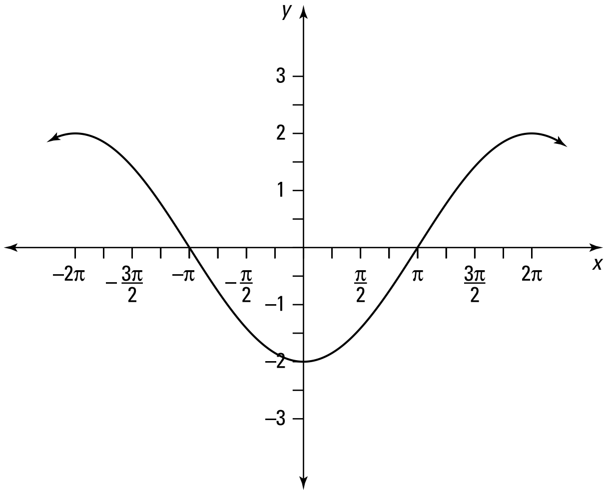
(B) $f(x) = 2\cos(2x)$

(C) $f(x) = 2\cos(\pi x)$

(D) $f(x) = -2\cos(2x)$

(E) $f(x) = 2\cos\left(\frac{\pi}{2}x\right)$

151.



(A) $f(x) = 2\cos(2x) + 1$

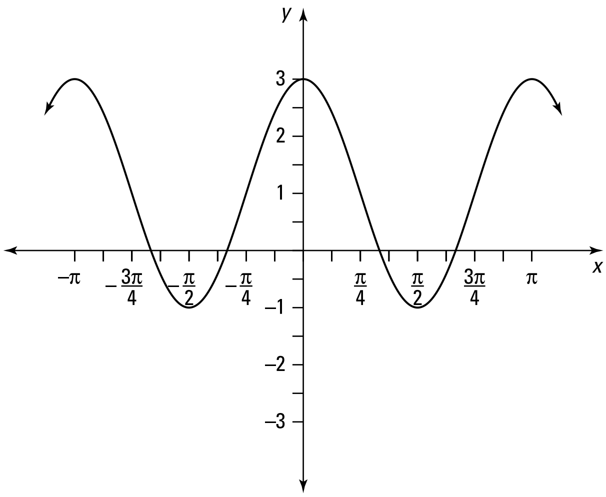
(B) $f(x) = -2\cos(2x) + 2$

(C) $f(x) = 2\cos(2x)$

(D) $f(x) = -2\cos\left(\frac{1}{2}x\right)$

(E) $f(x) = 2\cos(\pi x)$

152.



(A) $f(x) = -2\cos(2x)$

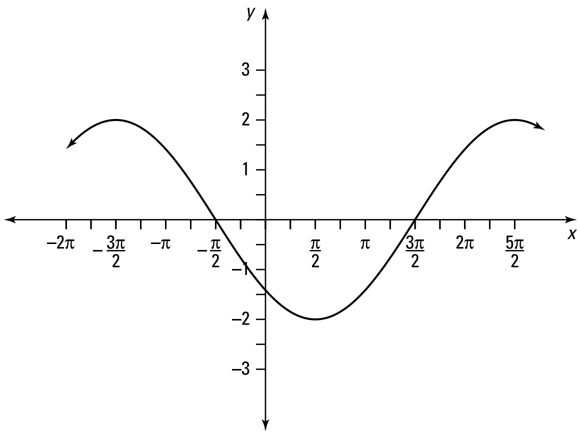
(B) $f(x) = -2\cos(2x) + 2$

(C) $f(x) = 2\cos(2x) + 1$

(D) $f(x) = 2\cos(\pi x) + 1$

(E) $f(x) = 2\cos\left(\frac{\pi}{2}x\right) + 1$

153.



(A) $f(x) = 2\cos\left(\frac{1}{2}x - \frac{\pi}{4}\right)$

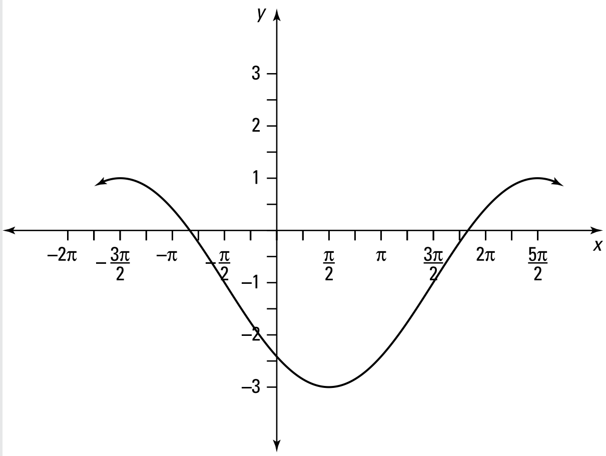
(B) $f(x) = 2\cos\left(\frac{1}{2}x - \frac{\pi}{2}\right)$

(C) $f(x) = -2\cos\left(\frac{1}{2}x - \frac{\pi}{4}\right)$

(D) $f(x) = -2\cos\left(x - \frac{\pi}{4}\right)$

(E) $f(x) = -2\cos\left(\frac{1}{2}x - \frac{\pi}{2}\right)$

154.



(A) $f(x) = 2\cos\left(\frac{1}{2}x - \frac{\pi}{4}\right) - 1$

(B) $f(x) = -2\cos\left(\frac{1}{2}x - \frac{\pi}{4}\right) + 1$

(C) $f(x) = -2\cos\left(x - \frac{\pi}{4}\right) - 1$

(D) $f(x) = -2\cos\left(\frac{1}{2}x - \frac{\pi}{4}\right) - 1$

(E) $f(x) = -2\cos\left(4x - \frac{\pi}{4}\right) - 1$

Inverse Trigonometric Function Basics

155–160 *Evaluate the inverse trigonometric function for the given value.*

155. Find the value of $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$.

156. Find the value of arctan(–1).

157. Find the value of $\cos\left(\sin^{-1}\left(\frac{1}{2}\right)\right)$.

158. Find the value of $\tan\left(\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)$.

159. Find the value of $\csc\left(\arccos\frac{4}{5}\right)$.

160. Find the value of $\sin\left(\tan^{-1}(2) + \tan^{-1}(3)\right)$.

Solving Trigonometric Equations Using Inverses

161–166 *Solve the given trigonometric equation using inverses. Find all solutions in the interval [0, 2π].*

161. $\sin x = 0.4$

162. $\cos x = -0.78$

163. $5\sin(2x) + 1 = 4$

164. $7\cos(3x) - 1 = 3$

165. $2\sin^2 x + 8\sin x + 5 = 0$

166. $3\sec^2 x + 4\tan x = 2$

Chapter 3

Limits and Rates of Change

Limits are the foundation of calculus. Being able to work with limits and to understand them conceptually is crucial, because key ideas and definitions in calculus make use of limits. This chapter examines a variety of limit problems and makes the intuitive idea of continuity formal by using limits. Many later problems also involve the use of limits, so although limits may go away for a while during your calculus studies, they'll return!

The Problems You'll Work On

In this chapter, you encounter a variety of problems involving limits:

- Using graphs to find limits
- Finding left-hand and right-hand limits
- Determining infinite limits and limits at infinity
- Practicing many algebraic techniques to evaluate limits of the form 0/0
- Determining where a function is continuous

What to Watch Out For

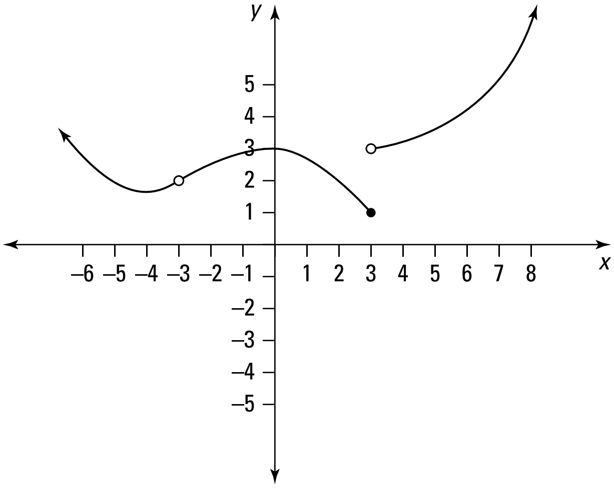
You can use a variety of techniques to evaluate limits, and you want to be familiar with them all! Remember the following tips:

- When substituting in the limiting value, a value of zero in the denominator of a fraction doesn't automatically mean that the limit does not exist! For example, if the function has a removable discontinuity, the limit still exists!
- Be careful with signs, as you may have to include a negative when evaluating limits at infinity involving radicals (especially when the variable approaches negative infinity). It's easy to make a limit positive when it should have been negative!
- Know and understand the definition of *continuity,* which says the following: A function $f(x)$ is continuous at a if $\lim_{x \to a} f(x) = f(a)$.

Finding Limits from Graphs

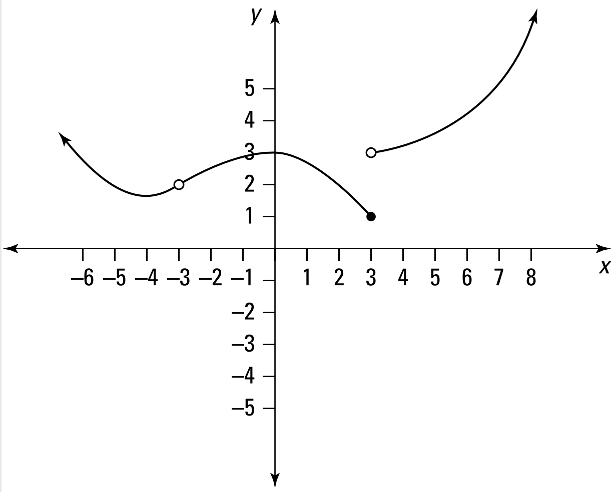
167–172 *Use the graph to find the indicated limit.*

167.



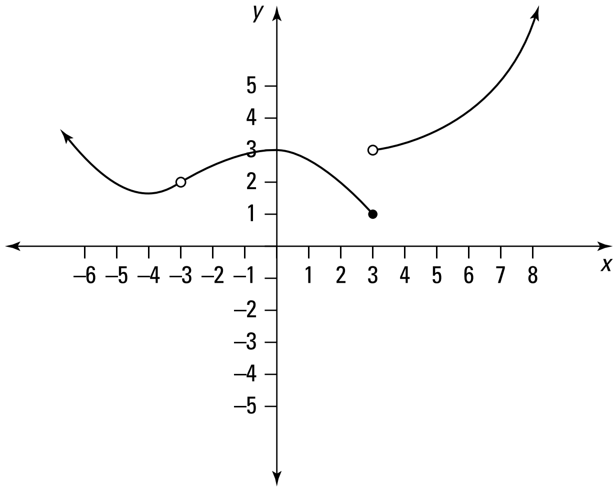
$\lim\limits_{x \to 3^-} f(x)$

168.



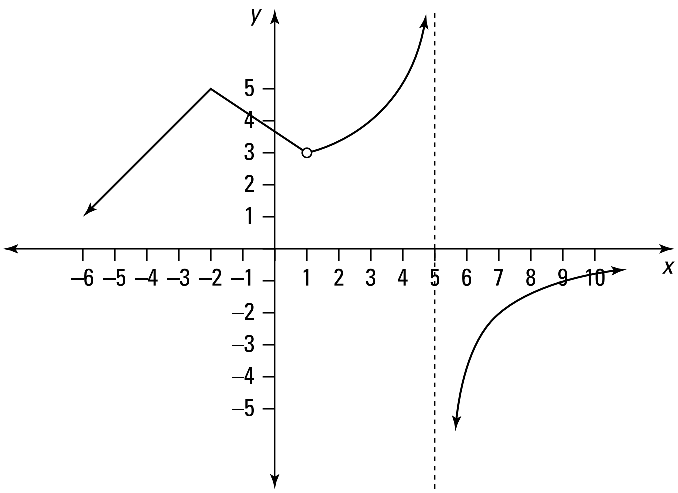
$\lim\limits_{x \to 3^+} f(x)$

169.



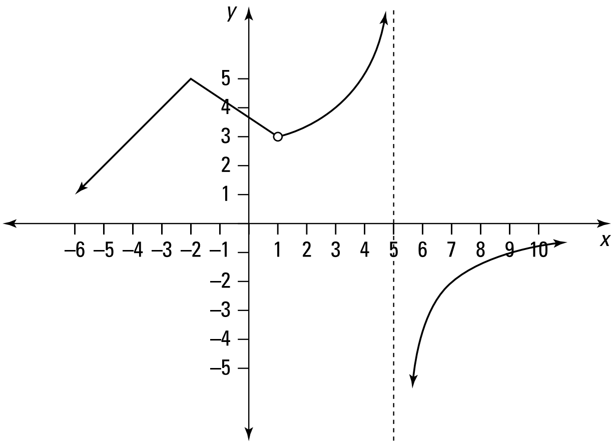
$\lim\limits_{x \to -3} f(x)$

170.



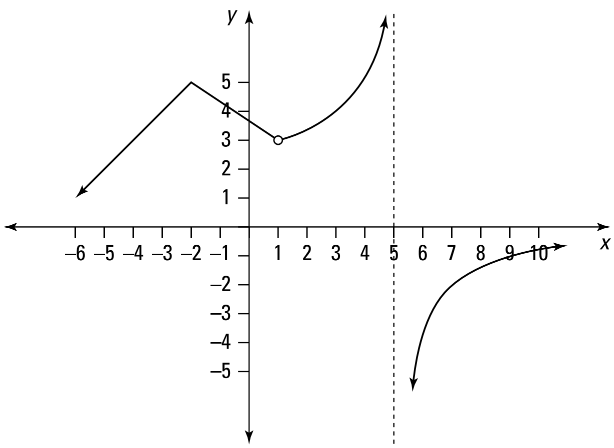
$\lim\limits_{x \to 1^-} f(x)$

171.



$\lim_{x \to 1^+} f(x)$

172.



$\lim_{x \to -2} f(x)$

Evaluating Limits

173–192 *Evaluate the given limit.*

173. $\lim_{x \to 3} \frac{x^2 - 2x - 3}{x - 3}$

174. $\lim_{x \to 2} \frac{x^2 + 3x - 10}{x^2 - 8x + 12}$

175. $\lim_{x \to -5} \frac{x^2 + 5x}{x^2 - 25}$

176. $\lim_{x \to 4} \frac{4 - x}{2 - \sqrt{x}}$

177. $\lim_{x \to 0} \left(\frac{1}{x} - \frac{1}{x^2 + x} \right)$

178. $\lim_{x \to 4} |x - 4|$

179. $\lim_{x \to -1} \frac{x^3 + 1}{x + 1}$

180. $\lim_{h \to 0} \frac{\sqrt{4 + h} - 2}{h}$

181. $\lim_{x\to 3} \frac{x^4 - 81}{x - 3}$

182. $\lim_{x\to 0^+} \left(\frac{2}{x^2} + \frac{2}{|x|} \right)$

183. $\lim_{x\to 0^-} \left(\frac{2}{x^2} - \frac{2}{|x|} \right)$

184. $\lim_{x\to \frac{4}{3}} \frac{3x^2 - 4x}{|3x - 4|}$

185. $\lim_{x\to 5} \frac{x^2 - 25}{3x^2 - 16x + 5}$

186. $\lim_{x\to 3} \frac{\sqrt{x+3} - \sqrt{2x}}{x^2 - 3x}$

187. $\lim_{h\to 0} \frac{(2+h)^3 - 8}{h}$

188. $\lim_{x\to 4^+} \frac{x - 4}{|x - 4|}$

189. $\lim_{x\to 5^-} \frac{x - 5}{|x - 5|}$

190. $\lim_{x\to -5} \frac{1/5 + 1/x}{x + 5}$

191. $\lim_{x\to 0} \left(\frac{1}{x\sqrt{1+x}} - \frac{1}{x} \right)$

192. $\lim_{h\to 0} \frac{(4+h)^{-1} - 4^{-1}}{h}$

Applying the Squeeze Theorem

193–198 *Use the squeeze theorem to evaluate the given limit.*

193. If $5 \le f(x) \le x^2 + 3x - 5$ for all x, find $\lim_{x\to 2} f(x)$.

194. If $x^2 + 4 \le f(x) \le 4 + \sin x$ for $-2 \le x \le 5$, find $\lim_{x\to 0} f(x)$.

195. If $2x \le f(x) \le x^3 + 1$ for $0 \le x \le 2$, evaluate $\lim_{x\to 1} f(x)$.

196. Find the limit: $\lim_{x\to 0} x^4 \cos\left(\frac{2}{x^2} \right)$.

197. Find the limit: $\lim_{x\to 0^+} x^2 \sin\left(\frac{2}{\sqrt{x}}\right)$.

198. Find the limit: $\lim_{x\to 0^+} \sqrt[3]{x}\left(3-\sin^2\left(\frac{\pi}{x}\right)\right)$.

Evaluating Trigonometric Limits

199–206 *Evaluate the given trigonometric limit. Recall that* $\lim_{x\to 0}\frac{\sin x}{x}=1$ *and that* $\lim_{x\to 0}\frac{\cos x-1}{x}=0$.

199. $\lim_{x\to 0}\frac{\sin(5x)}{x}$

200. $\lim_{x\to 0}\frac{2\cos x-2}{\sin x}$

201. $\lim_{x\to \frac{\pi}{4}}\frac{\cos 2x}{\sin x-\cos x}$

202. $\lim_{x\to 0}\frac{\sin(5x)}{\sin(9x)}$

203. $\lim_{x\to 0}\frac{\tan(7x)}{\sin(3x)}$

204. $\lim_{x\to 0}\frac{\sin^3(2x)}{x^3}$

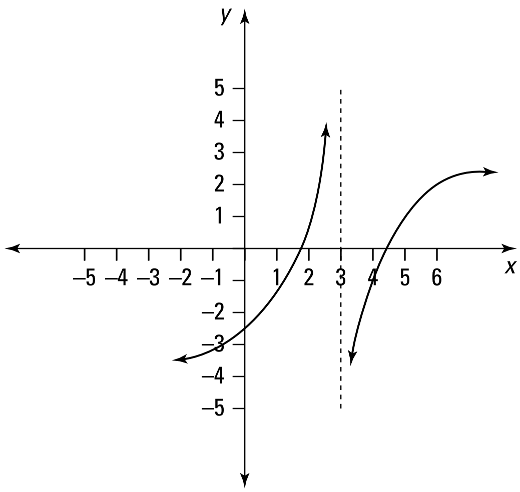
205. $\lim_{x\to 2}\frac{\sin(x-2)}{x^2+x-6}$

206. $\lim_{x\to 0}\frac{\sin x}{x+\tan x}$

Infinite Limits

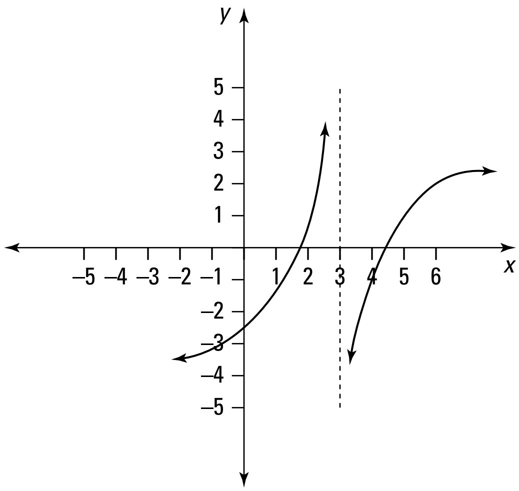
207–211 *Find the indicated limit using the given graph.*

207.



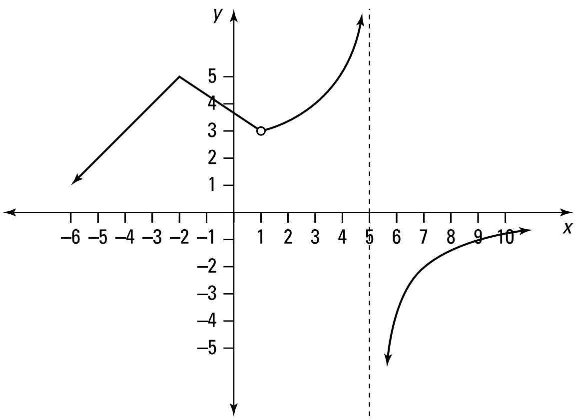
$\lim_{x\to 3^-} f(x)$

208.



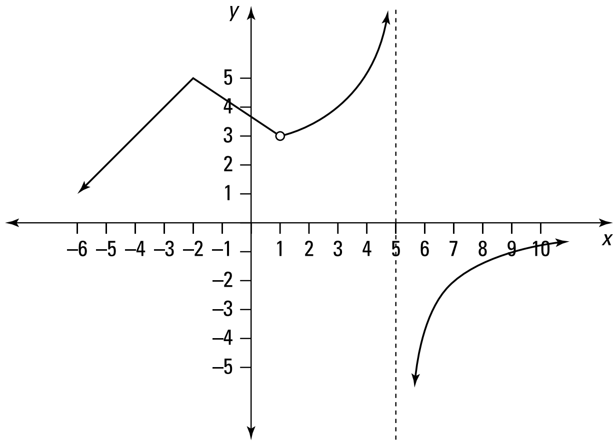
$\lim_{x \to 3^+} f(x)$

209.



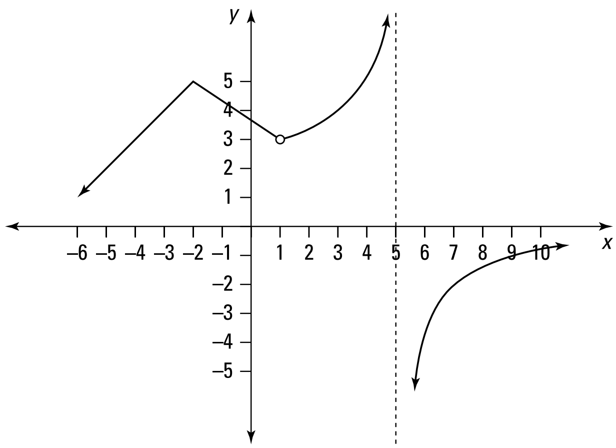
$\lim_{x \to 5^-} f(x)$

210.



$\lim_{x \to 5^+} f(x)$

211.



$\lim_{x \to 5} f(x)$

212–231 *Find the indicated limit.*

212. $\lim_{x\to 1^+} \frac{3}{x-1}$

213. $\lim_{x\to 1^-} \frac{3}{x-1}$

214. $\lim_{x\to \frac{\pi}{2}^+} (\tan x)$

215. $\lim_{x\to \pi^-} \frac{x^2}{\sin x}$

216. $\lim_{x\to 5^-} \frac{x+3}{x-5}$

217. $\lim_{x\to 0^-} \frac{1-x}{e^x-1}$

218. $\lim_{x\to 0^-} \cot x$

219. $\lim_{x\to 2} \frac{4e^x}{|2-x|}$

220. $\lim_{x\to \frac{1}{2}^+} \frac{x^2+1}{x\cos(\pi x)}$

221. $\lim_{x\to 5} \frac{\sin x}{(5-x)^4}$

222. $\lim_{x\to 0} \frac{x+5}{x^4(x-6)}$

223. $\lim_{x\to 1} \frac{3x}{e^x-e}$

224. $\lim_{x\to 0^+} \frac{x-1}{x^2(x+2)}$

225. $\lim_{x\to e^-} \frac{x^3}{\ln x-1}$

226. $\lim_{x\to e^2} \frac{-x}{\ln x-2}$

227. $\lim_{x\to 2} \frac{x+2}{x^2-4}$

228. $\lim_{x\to 25} \frac{5+\sqrt{x}}{x-25}$

229. $\lim_{x\to 0} \frac{x^2+4}{x^2(x-1)}$

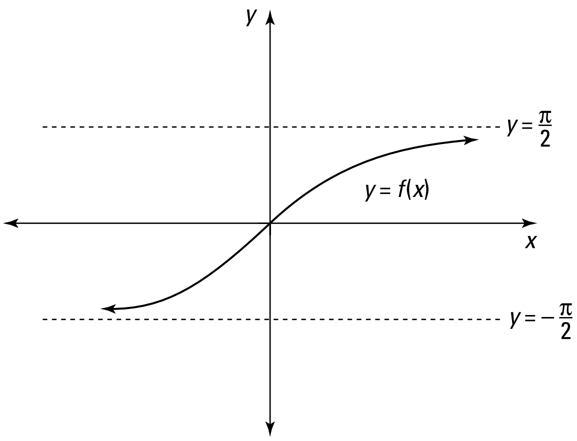
230. $\lim_{x \to 0^-} \frac{x-1}{x^2(x+2)}$

231. $\lim_{x \to 3} \frac{3+x}{|3-x|}$

Limits from Graphs

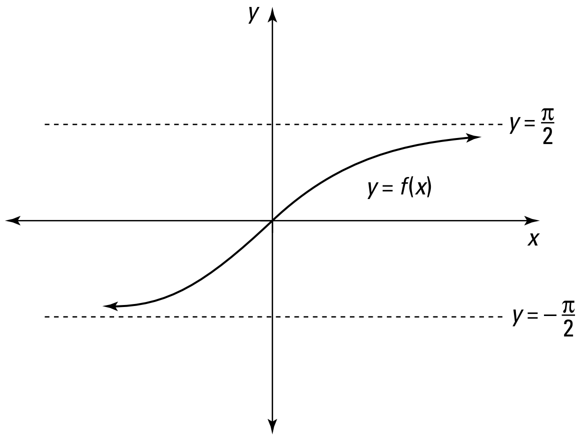
232–235 *Find the indicated limit using the given graph.*

232.



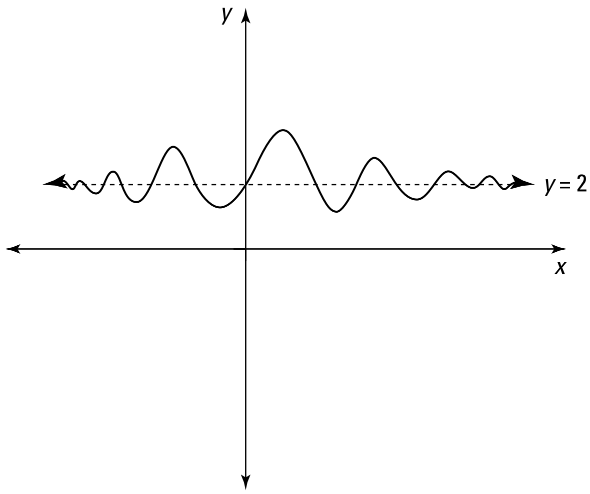
$\lim_{x \to -\infty} f(x)$

233.



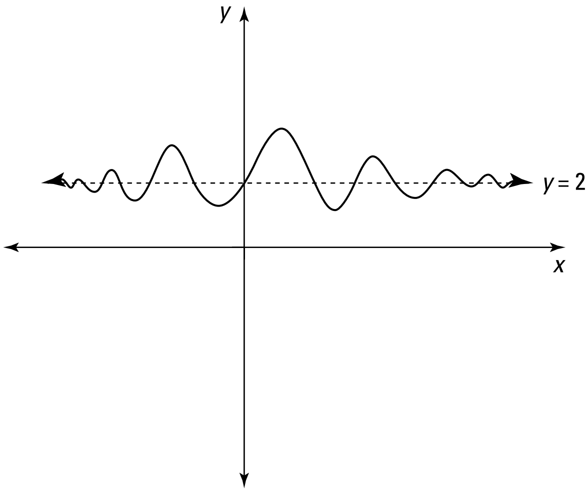
$\lim_{x \to \infty} f(x)$

234.



$\lim_{x \to -\infty} f(x)$

235.



$\lim_{x\to\infty} f(x)$

Limits at Infinity

236–247 *Find the indicated limit.*

236. $\lim_{x\to\infty}\frac{1}{3x+4}$

237. $\lim_{x\to-\infty}\left[x^2(x+1)(3-x)\right]$

238. $\lim_{x\to\infty}\frac{3x+4}{x-7}$

239. $\lim_{x\to\infty}\cos x$

240. $\lim_{x\to\infty}\frac{5x^4+5}{\left(x^2-1\right)\left(2x^3+3\right)}$

241. $\lim_{x\to-\infty}\frac{x}{\sqrt{x^2+1}}$

242. $\lim_{x\to-\infty}\frac{8x^4+3x-5}{x^2+1}$

243. $\lim_{x\to-\infty}\frac{\sqrt{9x^{10}-x}}{x^5+1}$

244. $\lim_{x\to\infty}\frac{\sqrt{9x^{10}-x}}{x^5+1}$

245. $\lim_{x\to\infty}\left(\sqrt{x}-x\right)$

246. $\lim_{x\to\infty}\left(\sqrt{x^4+3x^2}-x^2\right)$

247. $\lim_{x\to-\infty}\left(x+\sqrt{x^2+5x}\right)$

Horizontal Asymptotes

248–251 *Find any horizontal asymptotes of the given function.*

248. $y = \dfrac{1+3x^4}{x+5x^4}$

249. $y = \dfrac{5-x^2}{5+x^2}$

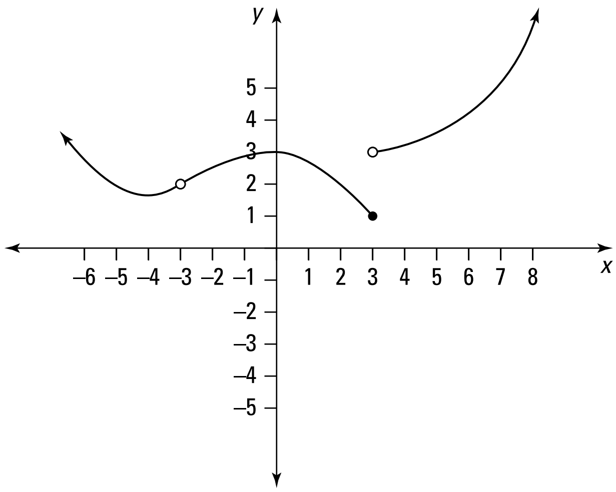
250. $y = \dfrac{\sqrt{x^4+x}}{3x^2}$

251. $y = \dfrac{x}{\sqrt{x^2+2}}$

Classifying Discontinuities

252–255 *Use the graph to find all discontinuities and classify each one as a jump discontinuity, a removable discontinuity, or an infinite discontinuity.*

252.



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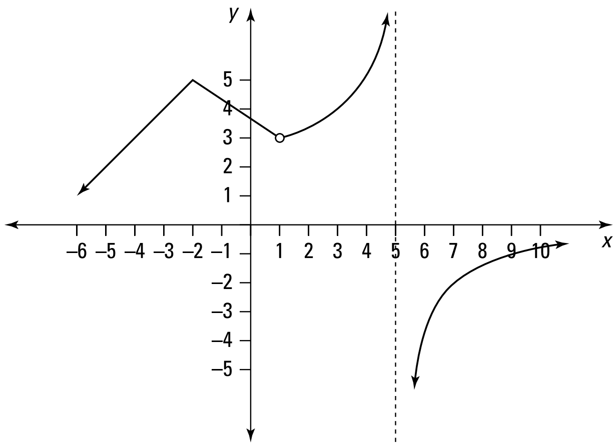

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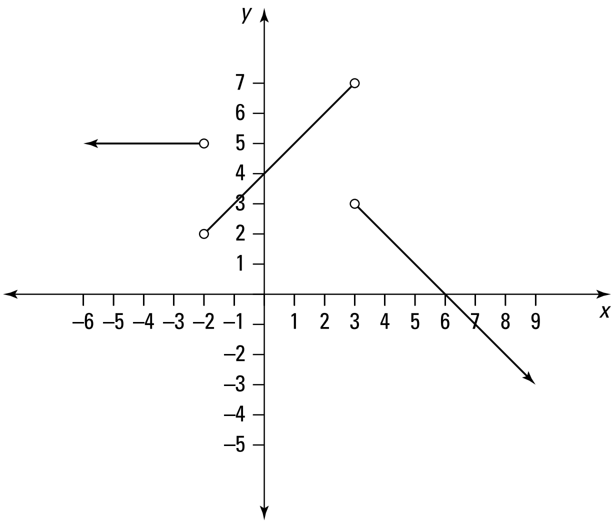

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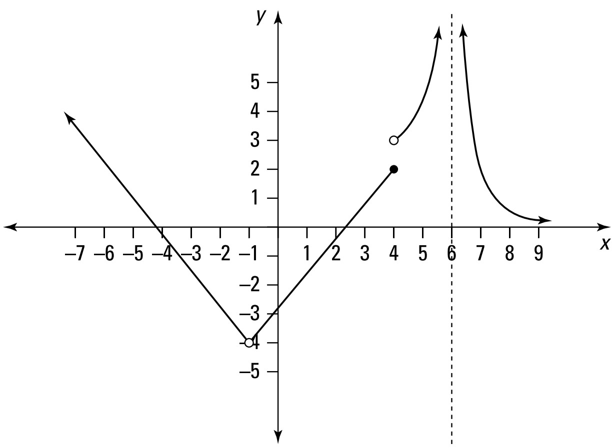


Continuity and Discontinuities

256–261 *Determine whether the function is continuous at the given value of a. If it's continuous, state the value at f(a). If it isn't continuous, classify the discontinuity as a jump, removable, or infinite discontinuity.*

256. $f(x)=\begin{cases}\dfrac{1}{x-2} & x\neq 2\\ 3 & x=2\end{cases}$

where $a = 2$

257. $f(x)=\begin{cases}1+x^2 & x\leq 1\\ 4\sqrt{x}-2 & x>1\end{cases}$

where $a = 1$

258. $f(x)=\begin{cases}\dfrac{x^2-x-6}{x-3} & x\neq 3\\ 5 & x=3\end{cases}$

where $a = 3$

259. $f(x)=\begin{cases}\dfrac{4-\sqrt{x}}{16-x} & x\neq 16\\ \dfrac{1}{8} & x=16\end{cases}$

where $a = 16$

260. $f(x)=\begin{cases} \dfrac{x+6}{|x+6|} & x \neq -6 \\ -1 & x=-6 \end{cases}$

where $a = -6$

261. $f(x)=\begin{cases} \dfrac{x^3+1}{x+1} & x \neq -1 \\ 2 & x=-1 \end{cases}$

where $a = -1$

262–265 *Determine whether the function is continuous at the given values of a. If it isn't continuous, classify each discontinuity as a jump, removable, or infinite discontinuity.*

262. $f(x)=\begin{cases} 2+x^2 & x \le 0 \\ 2\cos x & 0 < x \le \pi \\ \sin x - 2 & \pi < x \end{cases}$

where $a = 0$ and $a = \pi$

263. $f(x)=\begin{cases} x+2 & x \le 1 \\ 2x^2 & 1 < x \le 3 \\ x^3 & 3 < x \end{cases}$

where $a = 1$ and $a = 3$

264. $f(x)=\begin{cases} \sqrt{2-x} & x \le 2 \\ x^2-4 & 2 < x \le 3 \\ \dfrac{1}{x+5} & 3 < x \end{cases}$

where $a = 2$ and $a = 3$

265. $f(x)=\begin{cases} \cos x & x \le 0 \\ \dfrac{1}{x} & 0 < x \le 4 \\ \dfrac{2}{x+4} & 4 < x \end{cases}$

where $a = 0$ and $a = 4$

Making a Function Continuous

266–267 *Determine the value of c that makes the given function continuous everywhere.*

266. $f(x)=\begin{cases} cx-2 & x \le 2 \\ cx^2+1 & 2 < x \end{cases}$

267. $f(x)=\begin{cases} x^2+c^2 & x \le 4 \\ cx+12 & 4 < x \end{cases}$

The Intermediate Value Theorem

268–271 *Determine which of the given intervals is guaranteed to contain a root of the function by the intermediate value theorem.*

268. By checking only the endpoints of each interval, determine which interval contains a root of the function $f(x) = x^2 - \frac{3}{2}$ by the intermediate value theorem:

(A) [–5, –4]

(B) [–4, –3]

(C) [0, 1]

(D) [1, 2]

(E) [5, 12]

269. By checking only the endpoints of each interval, determine which interval contains a root of the function $f(x) = 3\sqrt{x} - 4x + 5$ by the intermediate value theorem:

(A) [0, 1]

(B) [1, 4]

(C) [4, 9]

(D) [9, 16]

(E) [16, 25]

270. By checking only the endpoints of each interval, determine which interval contains a solution to the equation $2(3^x) + x^2 - 4 = 32$ according to the intermediate value theorem:

(A) [0, 1]

(B) [1, 2]

(C) [2, 3]

(D) [3, 4]

(E) [4, 5]

271. By checking only the endpoints of each interval, determine which interval contains a solution to the equation $4|2x - 3| + 5 = 22$ according to the intermediate value theorem:

(A) [0, 1]

(B) [1, 2]

(C) [2, 3]

(D) [3, 4]

(E) [4, 5]

Chapter 4

Derivative Basics

The derivative is one of the great ideas in calculus. In this chapter, you see the formal definition of a derivative. Understanding the formal definition is crucial, because it tells you what a derivative actually *is*. Unfortunately, computing the derivative using the definition can be quite cumbersome and is often very difficult. After finding derivatives using the definition, you see problems that use the power rule, which is the start of some techniques that make finding the derivative much easier — although still challenging in many cases.

The Problems You'll Work On

In this chapter, you see the definition of a derivative and one of the first shortcut formulas, the power rule. Here's what the problems cover:

- Using a variety of algebraic techniques to find the derivative using the definition of a derivative
- Evaluating the derivative at a point using a graph and slopes of tangent lines
- Encountering a variety of derivative questions that you can solve using the power rule

What to Watch Out For

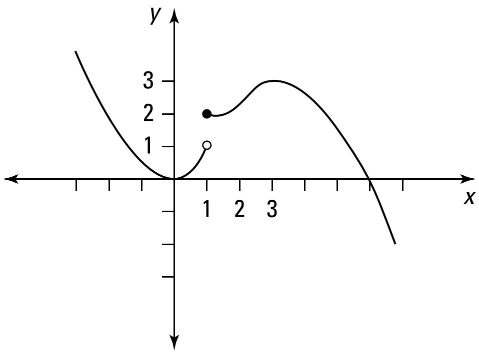
Using the definition of a derivative to evaluate derivatives can involve quite a bit of algebra, so be prepared. Having all the shortcut techniques is very nice, but you'll be asked to find derivatives for complicated functions, so the problems will still be challenging! Keep some of the following points in mind:

- Remember your algebra techniques: factoring, multiplying by conjugates, working with fractions, and more. Many students get tripped up on one part and then can't finish the problem, so know that many problems require multiple steps.
- When interpreting the value of a derivative from a graph, think about the slope of the tangent line on the graph at a given point; you'll be well on your way to finding the correct solution.
- Simplifying functions using algebra and trigonometric identities before finding the derivative makes many problems much easier. Simplifying is one of the very first things you should consider when encountering a "find the derivative" question of any type.

Determining Differentiability from a Graph

272–276 *Use the graph to determine for which values of x the function is not differentiable.*

272.



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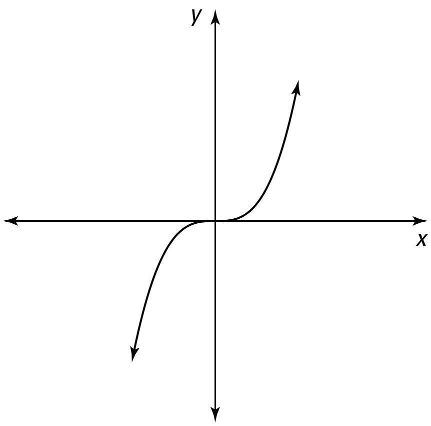

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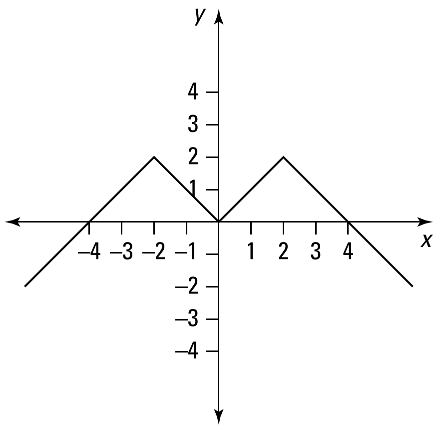

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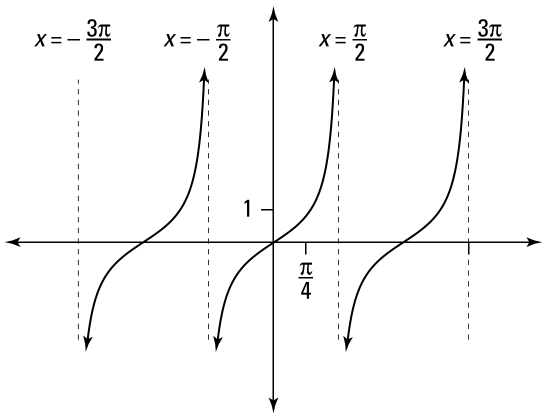

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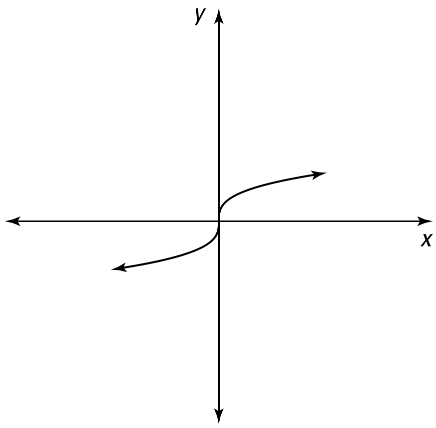


Finding the Derivative by Using the Definition

277–290 *Find the derivative by using the definition* $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$.

277. $f(x) = 2x - 1$

278. $f(x) = x^2$

279. $f(x) = 2 + x$

280. $f(x)=\frac{1}{x}$

281. $f(x) = x^3 - x^2$

282. $f(x) = 3x^2 + 4x$

283. $f(x)=\sqrt{x}$

284. $f(x)=\sqrt{2-5x}$

285. $f(x)=\frac{1}{\sqrt{x-1}}$

286. $f(x) = x^3 + 3x$

287. $f(x)=\frac{1}{x^2+5}$

288. $f(x)=\frac{2x+1}{x+4}$

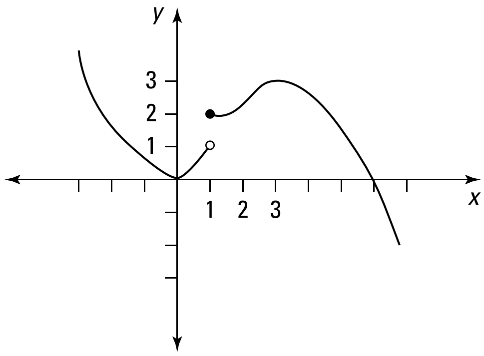
289. $f(x)=\frac{x^2+1}{x+3}$

290. $f(x)=\sqrt[3]{2x+1}$

Finding the Value of the Derivative Using a Graph

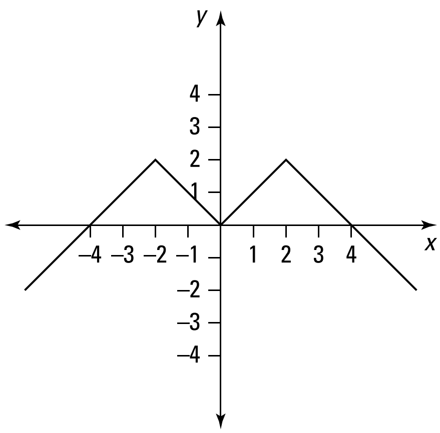
291–296 *Use the graph to determine the solution.*

291.



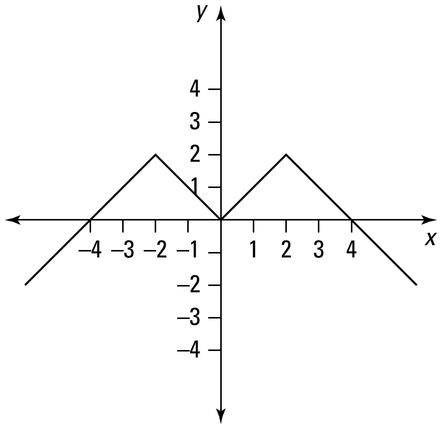
Estimate the value of $f'(3)$ using the graph.

292.



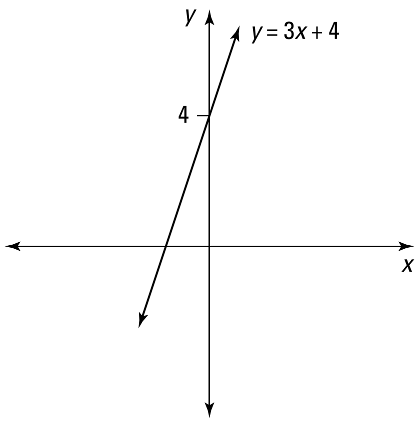
Estimate the value of $f'(-1)$ using the graph.

293.



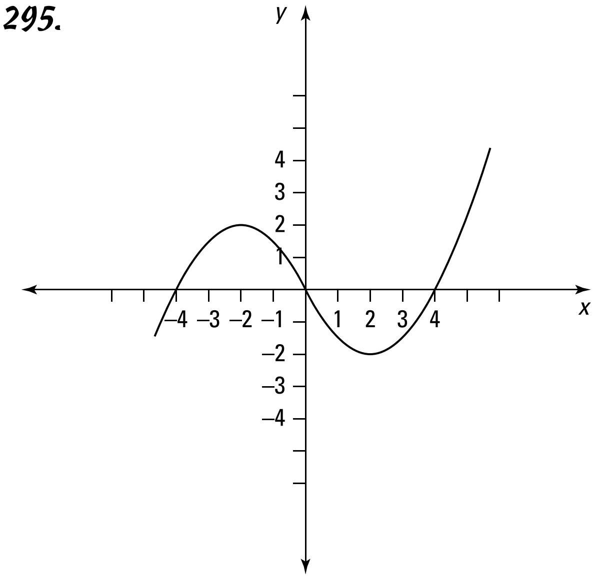
Estimate the value of $f'(-3)$ using the graph.

294.



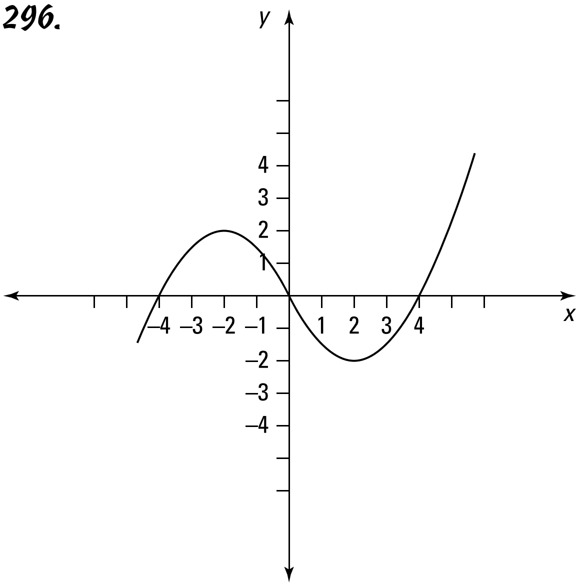
Based on the graph of $y = 3x + 4$, what does $f'(-22\pi^3)$ equal?

295.



Based on the graph, arrange the following from smallest to largest: $f'(-3)$, $f'(-2)$, and $f'(1)$.

296.



Based on the graph, arrange the following from smallest to largest: $f'(1)$, $f'(2)$, $f'(5)$, and 0.1.

Using the Power Rule to Find Derivatives

297–309 *Use the power rule to find the derivative of the given function.*

297. $f(x) = 5x + 4$

298. $f(x) = x^2 + 3x + 6$

299. $f(x) = (x + 4)(2x - 1)$

300. $f(x) = \pi^3$

301. $f(x) = \sqrt{5x}$

302. $f(x) = \dfrac{3x^2 + x - 4}{x^3}$

303. $f(x) = \sqrt{x}\left(x^3 + 1\right)$

304. $f(x) = \dfrac{\sqrt{3}}{x^4} + 2x^{1.1}$

305. $f(x) = 4x^{-3/7} + 8x + \sqrt{5}$

306. $f(x) = \left(\frac{1}{x^3} - \frac{2}{x}\right)\left(x^2 + x\right)$

307. $f(x) = (x^{-3} + 4)(x^{-2} - 5x)$

308. $f(x) = 4x^4 - x^2 + 8x + \pi^2$

309. $f(x) = \sqrt{x} - \frac{1}{\sqrt[4]{x}}$

Finding All Points on a Graph Where Tangent Lines Have a Given Value

310–311 *Find all points on the given function where the slope of the tangent line equals the indicated value.*

310. Find all x values where the function $f(x) = x^3 - x^2 - x + 1$ has a horizontal tangent line.

311. Find all x values where the function $f(x) = 6x^3 + 5x - 2$ has a tangent line with a slope equal to 6.

Chapter 5

The Product, Quotient, and Chain Rules

This chapter focuses on some of the major techniques needed to find the derivative: the product rule, the quotient rule, and the chain rule. By using these rules along with the power rule and some basic formulas (see Chapter 4), you can find the derivatives of most of the single-variable functions you encounter in calculus. However, after using the derivative rules, you often need many algebra steps to simplify the function so that it's in a nice final form, especially on problems involving the product rule or quotient rule.

The Problems You'll Work On

Here you practice using most of the techniques needed to find derivatives (besides the power rule):

- The product rule
- The quotient rule
- The chain rule
- Derivatives involving trigonometric functions

What to Watch Out For

Many of these problems require one calculus step and then many steps of algebraic simplification to get to the final answer. Remember the following tips as you work through the problems:

- Considering simplifying a function before taking the derivative. Simplifying before taking the derivative is almost always easier than finding the derivative and then simplifying.
- Some problems have functions without specified formulas in the questions; don't be thrown off! Simply proceed as you normally would on a similar example.
- Many people make the mistake of using the product rule when they should be using the chain rule. Stop and examine the function before jumping in and taking the derivative. Make sure you recognize whether the question involves a product or a composition (in which case you must use the chain rule).
- Rewriting the function by adding parentheses or brackets may be helpful, especially on problems that involve using the chain rule multiple times.

Using the Product Rule to Find Derivatives

312–331 *Use the product rule to find the derivative of the given function.*

312. $f(x) = (2x^3 + 1)(x^5 - x)$

313. $f(x) = x^2 \sin x$

314. $f(x) = \sec x \tan x$

315. $f(x) = \dfrac{x}{\cos x}$

316. $f(x) = 4x \csc x$

317. Find $(fg)'(4)$ if $f(4) = 3$, $f'(4) = 2$, $g(4) = -6$, and $g'(4) = 8$.

318. $f(x) = \dfrac{g(x)}{x^3}$

319. $f(x) = (\sec x)(x + \tan x)$

320. $f(x) = \left(x^2 + x\right)\csc x$

321. $f(x) = 4x^3 \sec x$

322. $f(x) = \dfrac{\cot x}{x^{1/2}}$

323. Assuming that g is a differentiable function, find an expression for the derivative of $f(x) = x^2 g(x)$.

324. Assuming that g is a differentiable function, find an expression for the derivative of $f(x) = \dfrac{1 + x^2 g(x)}{x}$.

325. Find $(fg)'(3)$ if $f(3) = -2$, $f'(3) = 4$, $g(3) = -8$, and $g'(3) = 7$.

326. $f(x) = x^2 \cos x \sin x$

327. Assuming that g is a differentiable function, find an expression for the derivative of $f(x)=\frac{2+xg(x)}{\sqrt{x}}$.

328. $f(x)=\left(\frac{1}{x^2}-\frac{2}{x}\right)(\tan x)$

329. $f(x)=\frac{x-2x\sqrt[3]{x}\cos x}{x^2}$

330. Assuming that g is a differentiable function, find an expression for the derivative of $f(x)=\frac{4g(x)}{x^5}$.

331. Assuming that g and h are differentiable functions, find an expression for the derivative of $f(x)=[g(x)h(x)]\sin x$.

Using the Quotient Rule to Find Derivatives

332–351 *Use the quotient rule to find the derivative.*

332. $f(x)=\frac{2x+1}{3x+4}$

333. $f(x)=\frac{2x}{5+x^2}$

334. $f(x)=\frac{\sin x-1}{\cos x+1}$

335. $f(x)=\frac{x}{x^5+x^3+1}$

336. Assuming that f and g are differentiable functions, find the value of $\left(\frac{f}{g}\right)'(4)$ if $f(4)=5$, $f'(4)=-7$, $g(4)=8$, and $g'(4)=4$.

337. $f(x)=\frac{x+2}{3x+5}$

338. $f(x)=\frac{x^2}{3x^2-2x+1}$

339. $f(x)=\frac{\sin x}{x^3}$

340. $f(x)=\frac{(x-1)(x+2)}{(x-3)(x+1)}$

341. $f(x)=\frac{\sec x}{1+\sec x}$

342. $f(x)=\dfrac{4x^2}{\sin x+\cos x}$

343. Assuming that f and g are differentiable functions, find the value of $\left(\dfrac{f}{g}\right)'(5)$ if $f(5)=-4$, $f'(5)=2$, $g(5)=-7$, and $g'(5)=-6$.

344. $f(x)=\dfrac{x^2}{1+\sqrt{x}}$

345. $f(x)=\dfrac{\sin x}{\cos x+\sin x}$

346. $f(x)=\dfrac{x^2}{1+\sqrt[3]{x}}$

347. $f(x)=\dfrac{\sqrt{x}+2}{\sqrt{x}+1}$

348. $f(x)=\dfrac{\tan x-1}{\sec x+1}$

349. $f(x)=\dfrac{x}{x+\dfrac{4}{x}}$

350. Assuming that g is a differentiable function, find an expression for the derivative of $f(x)=\dfrac{g(x)}{x^3}$.

351. Assuming that g is a differentiable function, find an expression for the derivative of $f(x)=\dfrac{\cos x}{g(x)}$.

Using the Chain Rule to Find Derivatives

352–370 *Use the chain rule to find the derivative.*

352. $f(x)=\left(x^2+3x\right)^{100}$

353. $f(x)=\sin(4x)$

354. $f(x)=\sqrt[3]{1+\sec x}$

355. $f(x)=\dfrac{1}{\left(x^2-x\right)^5}$

356. $f(x)=\csc\left(\dfrac{1}{x^2}\right)$

357. $f(x)=\left(x+\cos^2 x\right)^5$

358. $f(x)=\dfrac{\sin(\pi x)}{\cos(\pi x)+\sin(\pi x)}$

359. $f(x) = \cos(x \sin x)$

360. $f(x)=x\sin\sqrt[3]{x}$

361. $f(x)=\dfrac{x}{\sqrt{3-2x}}$

362. $f(x) = \sec^2 x + \tan^2 x$

363. $f(x)=\dfrac{x}{\sqrt{1+x^2}}$

364. $f(x)=\left(x^2+1\right)\sqrt[4]{x^3+5}$

365. $f(x)=\left(x^4-1\right)^3\left(x^5+1\right)^6$

366. $f(x)=\dfrac{(x-1)^3}{\left(x^2+x\right)^5}$

367. $f(x)=\sqrt{\dfrac{x+1}{x-1}}$, where $x > 1$

368. $f(x)=\sin\left(\sin\left(\sin\left(x^2\right)\right)\right)$

369. $f(x)=\sqrt[3]{x+\sqrt{x}}$

370. $f(x)=(1+5x)^4\left(2+x-x^2\right)^7$

More Challenging Chain Rule Problems

***371–376** Solve the problem related to the chain rule.*

371. Find all x values in the interval $[0, 2\pi]$ where the function $f(x) = 2\cos x + \sin^2 x$ has a horizontal tangent line.

372. Suppose that H is a function such that $H'(x)=\dfrac{2}{x}$ for $x > 0$. Find an expression for the derivative of $F(x)=\left[H(x)\right]^3$.

373. Let $F(x) = f(g(x))$, $g(2) = -2$, $g'(2) = 4$, $f'(2) = 5$, and $f'(-2) = 7$. Find the value of $F'(2)$.

374. Let $F(x) = f(f(x))$, $f(2) = -2$, $f'(2) = -5$, and $f'(-2) = 8$. Find the value of $F'(2)$.

375. Suppose that H is a function such that $H'(x) = \frac{2}{x}$ for $x > 0$. Find an expression for the derivative of $f(x) = H(x^3)$.

376. Let $F(x) = f(g(x))$, $g(4) = 6$, $g'(4) = 8$, $f'(4) = 2$, and $f'(6) = 10$. Find the value of $F'(4)$.

Chapter 6

Exponential and Logarithmic Functions and Tangent Lines

After becoming familiar with the derivative techniques of the power, product, quotient, and chain rules, you simply need to know basic formulas for different functions. In this chapter, you see the derivative formulas for exponential and logarithmic functions. Knowing the derivative formulas for logarithmic functions also makes it possible to use logarithmic differentiation to find derivatives.

In many examples and applications, finding either the tangent line or the normal line to a function at a point is desirable. This chapter arms you with all the derivative techniques, so you'll be in a position to find tangent lines and normal lines for many functions.

The Problems You'll Work On

In this chapter, you do the following types of problems:

- Finding derivatives of exponential and logarithmic functions with a variety of bases
- Using logarithmic differentiation to find a derivative
- Finding the tangent line or normal line at a point

What to Watch Out For

Although you're practicing basic formulas for exponential and logarithmic functions, you still use the product rule, quotient rule, and chain rule as before. Here are some tips for solving these problems:

- Using logarithmic differentiation requires being familiar with the properties of logarithms, so make sure you can expand expressions containing logarithms.
- If you see an exponent involving something other than just the variable x, you likely need to use the chain rule to find the derivative.
- The tangent line and normal line are perpendicular to each other, so the slopes of these lines are opposite reciprocals.

Derivatives Involving Logarithmic Functions

377–385 *Find the derivative of the given function.*

377. $f(x) = \ln(x)^2$

378. $f(x) = (\ln x)^4$

379. $f(x) = \ln\sqrt{x^2+5}$

380. $f(x) = \log_{10}\left(x+\sqrt{6+x^2}\right)$

381. $f(x) = \log_6\left|\frac{-1+2\sin x}{4+3\sin x}\right|$

382. $f(x) = \log_7\left(\log_8 x^5\right)$

383. $f(x) = \ln\left|\sec x + \tan x\right|$

384. $f(x) = -\frac{\sqrt{x^2+1}}{x} + \ln\left(x+\sqrt{x^2+1}\right)$

385. $f(x) = \frac{\log_5\left(x^2\right)}{(x+1)^3}$

Logarithmic Differentiation to Find the Derivative

386–389 *Use logarithmic differentiation to find the derivative.*

386. $f(x) = x^{\tan x}$

387. $f(x) = (\ln x)^{\cos x}$

388. $f(x) = \sqrt{\frac{x^2-4}{x^2+4}}$

389. $f(x) = \frac{(x-1)\sin x}{\sqrt{x+2}(x+4)^{3/2}}$

Finding Derivatives of Functions Involving Exponential Functions

390–401 *Find the derivative of the given function.*

390. $f(x) = e^{5x}$

391. $f(x) = e^{\sin x + x^4}$

392. $f(x) = (x^3 + 1)2^x$

393. $f(x) = \log_5 5^{x^2+3}$

394. $f(x) = e^{x^3}(\sin x + \cos x)$

395. $f(x) = 5^{\sqrt{x}} \sin x$

396. $f(x) = (4^{-x} + 4^x)^3$

397. $f(x) = \ln\left(\dfrac{1+e^x}{1-e^x}\right)$

398. $f(x) = \dfrac{6^x + x}{x^2 + 1}$

399. $f(x) = \dfrac{4}{e^x + e^{-x}}$

400. $f(x) = \dfrac{8^{x^2+1}}{\cos x}$

401. $f(x) = x6^{-5x}$

Finding Equations of Tangent Lines

402–404 *Find the equation of the tangent line at the given value.*

402. $f(x) = 3\cos x + \pi x$ at $x = 0$

403. $f(x) = x^2 - x + 2$ at $(1, 2)$

404. $f(x) = \dfrac{e^{x^2}}{x}$ at $x = 2$

Finding Equations of Normal Lines

405–407 *Find the equation of the normal line at the indicated point.*

405. $f(x) = 3x^2 + x - 2$ at $(3, 28)$

406. $f(x) = \sin^2 x$ at $x = \frac{\pi}{4}$

407. $f(x) = 4 \ln x + 2$ at $x = e^2$

Chapter 7
Implicit Differentiation

When you know the techniques of implicit differentiation (this chapter) and logarithmic differentiation (covered in Chapter 6), you're in a position to find the derivative of just about any function you encounter in a single-variable calculus course. Of course, you'll still use the power, product, quotient, and chain rules (Chapters 4 and 5) when finding derivatives.

The Problems You'll Work On

In this chapter, you use implicit differentiation to

- Find the first derivative and second derivative of an implicit function
- Find slopes of tangent lines at given points
- Find equations of tangent lines at given points

What to Watch Out For

Lots of numbers and variables are floating around in these examples, so don't lose your way:

- Don't forget to multiply by dy/dx at the appropriate moment! If you aren't getting the correct solution, look for this mistake.
- After finding the second derivative of an implicitly defined function, substitute in the first derivative in order to write the second derivative in terms of x and y.
- When you substitute the first derivative into the second derivative, be prepared to further simplify.

Using Implicit Differentiation to Find a Derivative

408–413 *Use implicit differentiation to find* $\frac{dy}{dx}$.

408. $x^2 + y^2 = 9$

409. $y^5 + x^2y^3 = 2 + x^2y$

410. $x^3y^3 + x\cos(y) = 7$

411. $\sqrt{x+y} = \cos\left(y^2\right)$

412. $\cot\left(\frac{y}{x}\right) = x^2 + y$

413. $\sec(xy) = \frac{y}{1+x^2}$

Using Implicit Differentiation to Find a Second Derivative

414–417 *Use implicit differentiation to find* $\frac{d^2y}{dx^2}$.

414. $8x^2 + y^2 = 8$

415. $x^5 + y^5 = 1$

416. $x^3 + y^3 = 5$

417. $\sqrt{x} + \sqrt{y} = 1$

Finding Equations of Tangent Lines Using Implicit Differentiation

418–422 *Find the equation of the tangent line at the indicated point.*

418. $x^2 + xy + y^2 = 3$ at (1, 1)

419. $3\left(x^2+y^2\right)^2 = 25\left(x^2-y^2\right)$ at (2, 1)

420. $x^2 + 2xy + y^2 = 1$ at (0, 1)

421. $\cos(xy) + x^2 = \sin y$ at $\left(1, \frac{\pi}{2}\right)$

422. $y^2(y^2 - 1) = x^2 \tan y$ at (0, 1)

Chapter 8

Applications of Derivatives

What good are derivatives if you can't do anything useful with them? Well, don't worry! There are tons and tons of useful applications involving derivatives. This chapter illustrates how calculus can help solve a variety of practical problems, including finding maximum and minimum values of functions, approximating roots of equations, and finding the velocity and acceleration of an object, just to name a few. Without calculus, many of these problems would be very difficult indeed!

The Problems You'll Work On

This chapter has a variety of applications of derivatives, including

- Approximating values of a function using linearization
- Approximating roots of equations using Newton's method
- Finding the optimal solution to a problem by finding a maximum or minimum value
- Determining how quantities vary in relation to each other
- Locating absolute and local maxima and minima
- Finding the instantaneous velocity and acceleration of an object
- Using Rolle's theorem and the mean value theorem

What to Watch Out For

This chapter presents a variety of applications and word problems, and you may have to be a bit creative when setting up some of the problems. Here are some tips:

- Think about what your variables represent in the optimization and related-rates problems; if you can't explain what they represent, start over!
- You'll have to produce equations in the related-rates and optimization problems. Getting started is often the most difficult part, so just dive in and try different things.
- Remember that *linearization* is just a fancy way of saying "tangent line."
- Although things should be set up nicely in most of the problems, note that Newton's method doesn't always work; its success depends on your starting value.

Finding and Evaluating Differentials

423–425 *Find the differential dy and then evaluate dy for the given values of x and dx.*

423. $y = x^2 - 4x$, $x = 3$, $dx = \frac{1}{6}$

424. $y = \frac{1}{x^2+1}$, $x = 1$, $dx = -0.1$

425. $y = \cos^2 x$, $x = \frac{\pi}{3}$, $dx = 0.02$

Finding Linearizations

426–428 *Find the linearization L(x) of the function at the given value of a.*

426. $f(x) = 3x^2$, $a = 1$

427. $f(x) = \cos x + \sin x$, $a = \frac{\pi}{2}$

428. $f(x) = \sqrt[3]{x^2 + x}$, $a = 2$

Using Linearizations to Estimate Values

429–431 *Estimate the value of the given number using a linearization.*

429. Estimate $7.96^{2/3}$ to the thousandths place.

430. Estimate $\sqrt{102}$ to the tenths place.

431. Estimate $\tan 46°$ to the thousandths place.

Understanding Related Rates

432–445 *Solve the related-rates problem. Give an exact answer unless otherwise stated.*

432. If V is the volume of a sphere of radius r and the sphere expands as time passes, find $\frac{dV}{dt}$ in terms of $\frac{dr}{dt}$.

433. A pebble is thrown into a pond, and the ripples spread in a circular pattern. If the radius of the circle increases at a constant rate of 1 meter per second, how fast is the area of the circle increasing when the radius is 4 meters?

434. If $y = x^4 + 3x^2 + x$ and $\frac{dx}{dt} = 4$, find $\frac{dy}{dt}$ when $x = 3$.

435. If $z^3 = x^2 - y^2$, $\frac{dx}{dt} = 3$, and $\frac{dy}{dt} = 2$, find $\frac{dz}{dt}$ when $x = 4$ and $y = 1$.

436. Two sides of a triangle are 6 meters and 8 meters in length, and the angle between them is increasing at a rate of 0.12 radians per second. Find the rate at which the area of the triangle is increasing when the angle between the sides is $\frac{\pi}{6}$. Round your answer to the nearest hundredth.

437. A ladder 8 feet long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 3 feet per second, how fast is the angle between the top of the ladder and the wall changing when the angle is $\frac{\pi}{3}$ radians?

438. The base of a triangle is increasing at a rate of 2 centimeters per minute, and the height is increasing at a rate of 4 centimeters per minute. At what rate is the area changing when $b = 20$ centimeters and $h = 32$ centimeters?

439. At noon, Ship A is 150 kilometers east of Ship B. Ship A is sailing west at 20 kilometers per hour, and Ship B is sailing north at 35 kilometers per hour. How quickly is the distance between them changing at 3 p.m.? Round your answer to the nearest hundredth.

440. A particle moves along the curve $y = \sqrt[3]{x} + 1$. As the particle passes through the point (8, 3), the x coordinate is increasing at a rate of 5 centimeters per second. How quickly is the distance from the particle to the origin changing at this point? Round your answer to the nearest hundredth.

441. Two people start walking from the same point. One person walks west at 2 miles per hour, and the other walks southwest (at an angle 45° south of west) at 4 miles per hour. How quickly is the distance between them changing after 40 minutes? Round your answer to the nearest hundredth.

442. A trough is 20 feet long, and its ends are isosceles triangles that are 5 feet across the top and have a height of 2 feet. If the trough is being filled with water at a rate of 8 cubic feet per minute, how quickly is the water level rising when the water is 1 foot deep?

443. An experimental jet is flying with a constant speed of 700 kilometers per hour. It passes over a radar station at an altitude of 2 kilometers and climbs at an angle of 45°. At what rate is the distance from the plane to the radar station increasing 2 minutes later? Round your answer to the hundredths place.

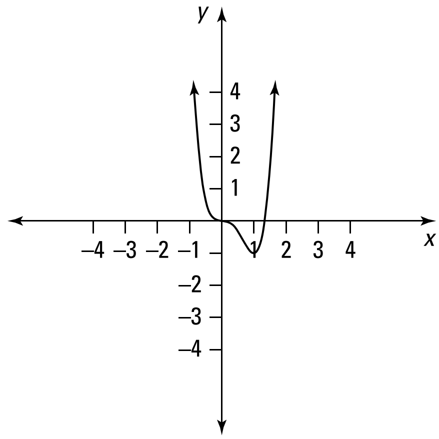
444. A lighthouse is located on an island 5 kilometers away from the nearest point P on a straight shoreline, and the light makes 6 revolutions per minute. How fast is the beam of light moving along the shore when it's 2 kilometers from P?

445. Gravel is being dumped into a pile that forms the shape of a cone whose base diameter is twice the height. If the gravel is being dumped at a rate of 20 cubic feet per minute, how fast is the height of the pile increasing when the pile is 12 feet high?

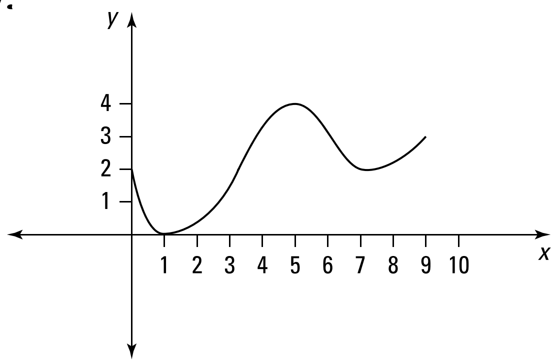
Finding Maxima and Minima from Graphs

446–450 *Use the graph to find the absolute maximum, absolute minimum, local maxima, and local minima, if any. Note that endpoints will not be considered local maxima or local minima.*

446.



447.



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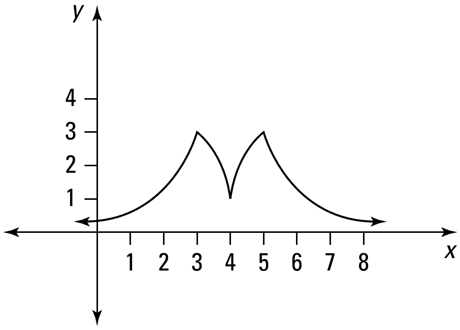

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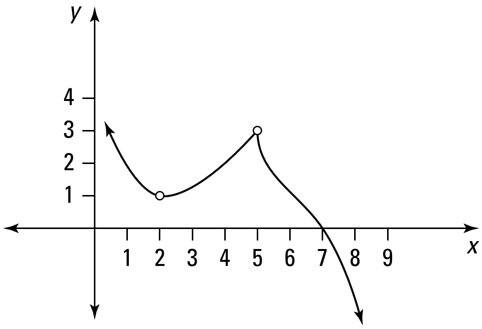

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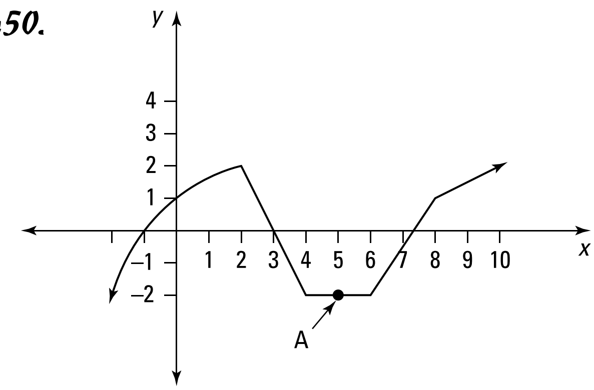


Point A corresponds to which of the following?

I. local maximum

II. local minimum

III. absolute maximum

IV. absolute minimum

Using the Closed Interval Method

451–455 *Find the absolute maximum and absolute minimum of the given function using the closed interval method.*

451. $f(x) = 3x^2 - 12x + 5$ on $[0, 3]$

452. $f(x) = x^4 - 2x^2 + 4$ on $[-2, 3]$

453. $f(x) = \dfrac{x}{x^2+1}$ on $[0, 3]$

454. $f(t) = t\sqrt{4-t^2}$ on $[-1, 2]$

455. $f(x) = x - 2\cos x$ on $[-\pi, \pi]$

Finding Intervals of Increase and Decrease

456–460 *Find the intervals of increase and decrease, if any, for the given function.*

456. $f(x) = 2x^3 - 24x + 1$

457. $f(x) = x\sqrt{x+3},\ x \geq -3$

458. $f(x) = \cos^2 x - \sin x$ on $[0, 2\pi]$

459. $f(x) = 2\cos x - \cos 2x$ on $0 \leq x \leq 2\pi$

460. $f(x) = 4\ln x - 2x^2$

Using the First Derivative Test to Find Local Maxima and Minima

461–465 *Use the first derivative to find any local maxima and any local minima.*

461. $f(x) = 2x^3 - 3x^2 - 12x$

462. $f(x) = x - 8\sqrt{x}$

463. $f(x) = 6x^{2/3} - x$

464. $f(x) = 2\sin x - \sin 2x$ on $[0, 2\pi]$

465. $f(x) = x + 2\cos x$ on $[-2\pi, 2\pi]$

Determining Concavity

466–470 *Find the intervals where the given function is concave up and concave down, if any.*

466. $f(x) = x^3 - 3x^2 + 4$

467. $f(x) = 9x^{2/3} - x$

468. $f(x) = x^{1/3}(x + 1)$

469. $f(x) = (x^2 - 4)^3$

470. $f(x) = 2\cos x - \sin(2x)$ on $[0, 2\pi]$

Identifying Inflection Points

471–475 *Find the inflection points of the given function, if any.*

471. $f(x) = \dfrac{1}{x^2 - 9} = \left(x^2 - 9\right)^{-1}$

472. $f(x) = 2x^3 + x^2$

473. $f(x) = \dfrac{\sin x}{1 + \cos x}$ on $[0, 2\pi]$

474. $f(x) = 3\sin x - \sin^3 x$ on $[0, 2\pi]$

475. $f(x) = x^{5/3} - 5x^{2/3}$

Using the Second Derivative Test to Find Local Maxima and Minima

476–480 *Use the second derivative test to find the local maxima and local minima of the given function.*

476. $f(x) = \sqrt[3]{\left(x^2 + 1\right)^2}$

477. $f(x) = x^4 - 4x^2 + 1$

478. $f(x) = 2x^2(1 - x^2)$

479. $f(x) = \dfrac{x}{x^2 + 4}$

480. $f(x) = 2\sin x - x$ on $[0, 2\pi]$

Applying Rolle's Theorem

481–483 *Verify that the function satisfies the hypotheses of Rolle's theorem. Then find all values c in the given interval that satisfy the conclusion of Rolle's theorem.*

481. $f(x) = x^2 - 6x + 1$, $[0, 6]$

482. $f(x) = x\sqrt{x + 8}$, $[-8, 0]$

483. $f(x) = \cos(2\pi x)$, $[-1, 1]$

Using the Mean Value Theorem

484–486 *Verify that the given function satisfies the hypotheses of the mean value theorem. Then find all numbers c that satisfy the conclusion of the mean value theorem.*

484. $f(x) = x^3 + 3x - 1$, $[0, 2]$

485. $f(x) = 2\sqrt[3]{x}$, $[0, 1]$

486. $f(x) = \frac{x}{x+2}$, $[1, 4]$

Applying the Mean Value Theorem to Solve Problems

487–489 *Solve the problem related to the mean value theorem.*

487. If $f(1) = 12$ and $f'(x) \geq 3$ for $1 \leq x \leq 5$, what is the smallest possible value of $f(5)$? Assume that f satisfies the hypothesis of the mean value theorem.

488. Suppose that $2 \leq f'(x) \leq 6$ for all values of x. What are the strictest bounds you can put on the value of $f(8) - f(4)$? Assume that f is differentiable for all x.

489. Apply the mean value theorem to the function $f(x) = x^{1/3}$ on the interval $[8, 9]$ to find bounds for the value of $\sqrt[3]{9}$.

Relating Velocity and Position

490–492 *Use the position function s(t) to find the velocity and acceleration at the given value of t. Recall that velocity is the change in position with respect to time and acceleration is the change in velocity with respect to time.*

490. $s(t) = t^2 - 8t + 4$ at $t = 5$

491. $s(t) = 2\sin t - \cos t$ at $t = \frac{\pi}{2}$

492. $s(t) = \frac{2t}{t^2+1}$ at $t = 1$

Finding Velocity and Speed

493–497 *Solve the given question related to speed or velocity. Recall that velocity is the change in position with respect to time.*

493. A mass on a spring vibrates horizontally with an equation of motion given by $x(t) = 8\sin(2t)$, where x is measured in feet and t is measured in seconds. Is the spring stretching or compressing at $t = \frac{\pi}{3}$? What is the speed of the spring at that time?

494. A stone is thrown straight up with the height given by the function $s = 40t - 16t^2$, where s is measured in feet and t is measured in seconds. What is the maximum height of the stone? What is the velocity of the stone when it's 20 feet above the ground on its way up? And what is its velocity at that height on the way down? Give exact answers.

495. A stone is thrown vertically upward with the height given by $s = 20t - 16t^2$, where s is measured in feet and t is measured in seconds. What is the maximum height of the stone? What is the velocity of the stone when it hits the ground?

496. A particle moves on a vertical line so that its coordinate at time t is given by $y = t^3 - 4t + 5$ for $t \geq 0$. When is the particle moving upward, and when is it moving downward? Give an exact answer in interval notation.

497. A particle moves on a vertical line so that its coordinate at time t is given by $y = 4t^2 - 6t - 2$ for $t \geq 0$. When is the particle moving upward, and when it is it moving downward? Give your answer in interval notation.

Solving Optimization Problems

498–512 *Solve the given optimization problem. Recall that a maximum or minimum value occurs where the derivative is equal to zero, where the derivative is undefined, or at an endpoint (if the function is defined on a closed interval). Give an exact answer, unless otherwise stated.*

498. Find two numbers whose difference is 50 and whose product is a minimum.

499. Find two positive numbers whose product is 400 and whose sum is a minimum.

500. Find the dimensions of a rectangle that has a perimeter of 60 meters and whose area is as large as possible.

501. Suppose a farmer with 1,500 feet of fencing encloses a rectangular area and divides it into four pens with fencing parallel to one side. What is the largest possible total area of the four pens?

502. A box with an open top is formed from a square piece of cardboard that is 6 feet wide. Find the largest volume of the box that can be made from the cardboard.

503. A box with an open top and a square base must have a volume of 16,000 cubic centimeters. Find the dimensions of the box that minimize the amount of material used.

504. Find the point(s) on the ellipse $8x^2 + y^2 = 8$ farthest from (1, 0).

505. Find the point on the line $y = 4x + 6$ that is closest to the origin.

506. A rectangular poster is to have an area of 90 square inches with 1-inch margins at the bottom and sides and a 3-inch margin at the top. What dimensions give you the largest printed area?

507. At which x values on the curve $f(x) = 2 + 20x^3 - 4x^5$ does the tangent line have the largest slope?

508. A rectangular storage container with an open top is to have a volume of 20 cubic meters. The length of the base is twice the width. The material for the base costs $20 per square meter. The material for the sides costs $12 per square meter. Find the cost of the materials for the cheapest such container. Round your answer to the nearest cent.

509. A piece of wire that is 20 meters long is cut into two pieces. One is shaped into a square, and the other is shaped into an equilateral triangle. How much wire should you use for the square so that the total area is at a maximum?

510. A piece of wire that is 20 meters long is cut into two pieces. One is bent into a square, and the other is bent into an equilateral triangle. How much wire should you use for the square so that the total area is at a minimum?

511. The illumination of a light source is directly proportional to the strength of the light source and inversely proportional to the square of the distance from the source. Two light sources, one five times as strong as the other, are placed 20 feet apart, and an object is placed on the line between them. How far from the bright light source should the object be placed so that the object receives the least illumination?

512. Find the area of the largest rectangle that can be inscribed in the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$.

Doing Approximations Using Newton's Method

***513–515** Find the fifth approximation of the root of the equation using the given first approximation.*

513. $x^3 + 4x - 4 = 0$ using $x_1 = 1$. Round the solution to the fifth decimal place.

514. $x^4 - 18 = 0$ using $x_1 = 2$. Round the solution to the seventh decimal place.

515. $x^5 + 3 = 0$ using $x_1 = -1$. Round the answer to the seventh decimal place.

Approximating Roots Using Newton's Method

***516–518** Find the root using Newton's method.*

516. Use Newton's method to find the root of $\cos x = x$ correct to five decimal places.

517. Use Newton's method to find the root of $x^3 - x^2 - 2$ in the interval [1, 2] correct to five decimal places.

518. Use Newton's method to find the positive root of $\sqrt{x+1} = x^2$ correct to five decimal places.

Chapter 9

Areas and Riemann Sums

This chapter provides some of the groundwork and motivation for antiderivatives. Finding the area underneath a curve has real-world applications; however, for many curves, finding the area is difficult if not impossible to do using simple geometry. Here, you approximate the area under a curve by using rectangles and then turn to Riemann sums. The problems involving Riemann sums can be quite long and involved, especially because shortcuts to finding the solution do exist; however, the approach used in Riemann sums is the same approach you use when tackling definite integrals. It's worth understanding the idea behind Riemann sums so you can apply that approach to other problems!

The Problems You'll Work On

This chapter presents the following types of problems:

- Using left endpoints, right endpoints, and midpoints to estimate the area underneath a curve
- Finding an expression for the definite integral using Riemann sums
- Expressing a given Riemann sum as a definite integral
- Evaluating definite integrals using Riemann sums

What to Watch Out For

Here are some things to keep in mind as you do the problems in this chapter:

- Estimating the area under a curve typically involves quite a bit of arithmetic but shouldn't be too difficult conceptually. The process should be straightforward after you do a few problems.
- The problems on expressing a given Riemann sum as a definite integral don't always have unique solutions.
- To evaluate the problems involving Riemann sums, you need to know a few summation formulas. You can find them in any standard calculus text if you don't remember them — or you can derive them!

Calculating Riemann Sums Using Left Endpoints

519–522 *Find the Riemann sum for the given function with the specified number of intervals using left endpoints.*

519. $f(x) = 2 + x^2$, $0 \le x \le 2$, $n = 4$

520. $f(x) = \sqrt[3]{x} + x$, $1 \le x \le 4$, $n = 5$. Round your answer to two decimal places.

521. $f(x) = 4 \ln x + 2x$, $1 \le x \le 4$, $n = 7$. Round your answer to two decimal places.

522. $f(x) = e^{3x} + 4$, $1 \le x \le 9$, $n = 8$. Give your answer in scientific notation, rounded to three decimal places.

Calculating Riemann Sums Using Right Endpoints

523–526 *Find the Riemann sum for the given function with the specified number of intervals using right endpoints.*

523. $f(x) = 1 + 2x$, $0 \le x \le 4$, $n = 4$

524. $f(x) = x \sin x$, $2 \le x \le 6$, $n = 5$. Round your answer to two decimal places.

525. $f(x) = \sqrt{x} - 1$, $0 \le x \le 5$, $n = 6$. Round your answer to two decimal places.

526. $f(x) = \dfrac{x}{x+1}$, $1 \le x \le 3$, $n = 8$. Round your answer to two decimal places.

Calculating Riemann Sums Using Midpoints

527–530 *Find the Riemann sum for the given function with the specified number of intervals using midpoints.*

527. $f(x) = 2\cos x$, $0 \le x \le 3$, $n = 4$. Round your answer to two decimal places.

528. $f(x) = \frac{\sin x}{x+1}$, $1 \le x \le 5$, $n = 5$. Round your answer to two decimal places.

529. $f(x) = 3e^x + 2$, $1 \le x \le 4$, $n = 6$. Round your answer to two decimal places.

530. $f(x) = \sqrt{x} + x$, $1 \le x \le 5$, $n = 8$. Round your answer to two decimal places.

Using Limits and Riemann Sums to Find Expressions for Definite Integrals

531–535 *Find an expression for the definite integral using the definition. Do not evaluate.*

531. $\int_1^4 \sqrt{x}\,dx$

532. $\int_0^\pi \sin^2 x\,dx$

533. $\int_1^5 \left(x^2 + x\right)dx$

534. $\int_0^{\pi/4} (\tan x + \sec x)\,dx$

535. $\int_4^6 \left(3x^3 + x^2 - x + 5\right)dx$

Finding a Definite Integral from the Limit and Riemann Sum Form

536–540 *Express the limit as a definite integral. Note that the solution is not necessarily unique.*

536. $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{3}{n}\left(4+\frac{3i}{n}\right)^6$

537. $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{\pi}{3n}\sec\left(\frac{i\pi}{3n}\right)$

538. $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{n}\left(6+\frac{i}{n}\right)$

539. $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{\pi}{2n}\left(\cos\left(\frac{i\pi}{2n}\right)+\sin\left(\frac{i\pi}{2n}\right)\right)$

540. $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{5}{n}\sqrt{\frac{5i}{n}+\frac{125i^3}{n^3}}$

Using Limits and Riemann Sums to Evaluate Definite Integrals

541–545 *Use the limit form of the definition of the integral to evaluate the integral.*

541. $\int_0^2(1+2x)\,dx$

542. $\int_0^4\left(1+3x^3\right)dx$

543. $\int_1^4(4-x)\,dx$

544. $\int_0^3\left(2x^2-x-4\right)dx$

545. $\int_1^3\left(x^2+x-5\right)dx$

Chapter 10

The Fundamental Theorem of Calculus and the Net Change Theorem

Using Riemann sums to evaluate definite integrals (see Chapter 9) can be a cumbersome process. Fortunately, the fundamental theorem of calculus gives you a much easier way to evaluate definite integrals. In addition to evaluating definite integrals in this chapter, you start finding *antiderivatives,* or indefinite integrals. The net change theorem problems at the end of this chapter offer some insight into the use of definite integrals.

Although the antiderivative problems you encounter in this chapter aren't too complex, finding antiderivatives is in general a much more difficult process than finding derivatives, so consider yourself warned! You encounter many challenging antiderivative problems in later chapters.

The Problems You'll Work On

In this chapter, you see a variety of antiderivative problems:

- Finding derivatives of integrals
- Evaluating definite integrals
- Computing indefinite integrals
- Using the net change theorem to interpret definite integrals and to find the distance and displacement of a particle

What to Watch Out For

Although many of the problems in the chapter are easier antiderivative problems, you still need to be careful. Here are some tips:

- Simplify before computing the antiderivative. Don't forget to use trigonometric identities when simplifying the integrand.
- You don't often see problems that ask you to find derivatives of integrals, but make sure you practice them. They usually aren't that difficult, so they make for easier points on a quiz or test.
- Note the difference between distance and displacement; distance is always greater than or equal to zero, whereas displacement may be positive, negative, or zero! Finding the distance traveled typically involves more work than simply finding the displacement.

Using the Fundamental Theorem of Calculus to Find Derivatives

546–557 *Find the derivative of the given function.*

546. $f(x) = \int_0^x \sqrt{1+4t}\,dt$

547. $f(x) = \int_3^x \left(2+t^6\right)^4 dt$

548. $f(x) = \int_0^x t^3 \cos(t)dt$

549. $f(x) = \int_x^4 e^{t^2} dt$

550. $f(x) = \int_{\sin x}^0 \left(1-t^2\right)dt$

551. $f(x) = \int_{\ln\left(x^2+1\right)}^4 e^t\,dt$

552. $f(x) = \int_{\cos x}^1 \left(t^2+\sin t\right)dt$

553. $f(x) = \int_{1/x}^1 \sin^2(t)dt$

554. $f(x) = \int_1^{x^3} \frac{1}{t+t^4}dt$

555. $f(x) = \int_{\tan x}^{x^3} \frac{1}{\sqrt{3+t}}dt$

556. $f(x) = \int_{2x}^{6x} \frac{t^2-1}{t^4+1}dt$

557. $f(x) = \int_{\log_5 x}^{x^2} 5^t\,dt$

Working with Basic Examples of Definite Integrals

558–570 *Evaluate the definite integral using basic antiderivative rules.*

558. $\int_1^3 5\,dx$

559. $\int_0^{\pi/4} \cos x\,dx$

560. $\int_0^{\pi/4} \sec^2 t\, dt$

561. $\int_0^{\pi/3} \sec x \tan x\, dx$

562. $\int_1^2 4^x\, dx$

563. $\int_0^{\pi} (x - \sin x)\, dx$

564. $\int_1^4 (x + x^3)\, dx$

565. $\int_1^8 2\sqrt[3]{x}\, dx$

566. $\int_{\pi/4}^{\pi/2} (\csc^2 x - 1)\, dx$

567. $\int_0^2 (1 + 2y - 4y^3)\, dy$

568. $\int_0^1 (\sqrt{x} + x)\, dx$

569. $\int_{-3}^{\pi/2} f(x)dx$, where

$$f(x) = \begin{cases} x, & -3 \le x \le 0 \\ \cos x, & 0 < x \le \frac{\pi}{2} \end{cases}$$

570. $\int_{-3}^{4} |x - 2|\, dx$

Understanding Basic Indefinite Integrals

571–610 *Find the indicated antiderivative.*

571. $\int x^{3/4} dx$

572. $\int \frac{5}{x^7}\, dx$

573. $\int (x^3 + 3x^2 - 1)\, dx$

574. $\int (3\cos x - 4\sin x)\,dx$

575. $\int \left(4\cos^2 x + 4\sin^2 x\right)dx$

576. $\int \tan^2 x\,dx$

577. $\int \left(3x^2 + 2x + 1\right)dx$

578. $\int \left(\frac{2}{3}x^{4/3} + 5x^4\right)dx$

579. $\int \left(5 + x + \tan^2 x\right)dx$

580. $\int 6\sqrt{x}\,\sqrt[3]{x}\,dx$

581. $\int \sqrt{5x}\,dx$

582. $\int \sqrt[6]{\frac{4}{x}}\,dx$

583. $\int \frac{1 - \sin^2 x}{3\cos^2 x}\,dx$

584. $\int x^2\left(x^2 + 3x\right)dx$

585. $\int \frac{x^2 + x + 1}{x^4}\,dx$

586. $\int \left(1 + x^2\right)^2 dx$

587. $\int \left(2 - \cot^2 x\right)dx$

588. $\int \frac{\sqrt{x} - 4x^2}{x}\,dx$

589. $\int \left(\sqrt[3]{x} + 1\right)^2 dx$

590. $\int \sqrt{x}\left(1+\frac{1}{\sqrt[3]{x}}\right)dx$

591. $\int \frac{\cos x}{\sin^2 x}\,dx$

592. $\int \frac{x+5x^6}{x^3}\,dx$

593. $\int \frac{x+1}{\sqrt[7]{x}}\,dx$

594. $\int \frac{1+\sin^2 x}{\sin^2 x}\,dx$

595. $\int (1-x)(2+x)\,dx$

596. $\int \sec x(\sec x-\cos x)\,dx$

597. $\int \frac{x^2-5x+6}{x-2}\,dx$

598. $\int \frac{x^3+1}{x+1}\,dx$

599. $\int \left(4x^{1.6}-x^{2.8}\right)dx$

600. $\int \frac{5+x}{x^{2.1}}\,dx$

601. $\int \left(x^{7/2}-x^{1/2}+\frac{1}{x^{1/5}}\right)dx$

602. $\int \frac{\sin 2x}{4\cos x}\,dx$

603. $\int \sin^2 x\left(1+\cot^2 x\right)dx$

604. $\int \left(\frac{4x^3+5x-2}{x^3}\right)dx$

605. $\int \left(3\sin^2 x+3\cos^2 x+4\right)dx$

606. $\int \frac{x^3 - 25x}{x+5}\,dx$

607. $\int \frac{\tan^2 x}{\sin^2 x}\,dx$

608. $\int \frac{\cos 2x}{\cos x + \sin x}\,dx$

609. $\int \frac{\cos 2x}{\sin^2 x}\,dx$

610. $\int \frac{\cos x + \cos x \tan^2 x}{\sec^2 x}\,dx$

Understanding the Net Change Theorem

611–619 *Use the net change theorem to interpret the given definite integral.*

611. If $w'(t)$ is the rate of a baby's growth in pounds per week, what does $\int_0^2 w'(t)dt$ represent?

612. If $r(t)$ represents the rate at which oil leaks from a tanker in gallons per minute, what does $\int_0^{180} r(t)dt$ represent?

613. A new bird population that is introduced into a refuge starts with 100 birds and increases at a rate of $p'(t)$ birds per month. What does $100 + \int_0^6 p'(t)\,dt$ represent?

614. If $v(t)$ is the velocity of a particle in meters per second, what does $\int_0^{10} v(t)dt$ represent?

615. If $a(t)$ is the acceleration of a car in meters per second squared, what does $\int_3^5 a(t)dt$ represent?

616. If $P'(t)$ represents the rate of production of solar panels, where t is measured in weeks, what does $\int_2^4 P'(t)\,dt$ represent?

617. The current in a wire $I(t)$ is defined as the derivative of the charge, $Q'(t) = I(t)$. What does $\int_{t_1}^{t_2} I(t)dt$ represent if t is measured in hours?

618. $I'(t)$ represents the rate of change in your income in dollars from a new job, where t is measured in years. What does $\int_0^{10} I'(t)dt$ represent?

619. Water is flowing into a pool at a rate of $w'(t)$, where t is measured in minutes and $w'(t)$ is measured in gallons per minute. What does $\int_{60}^{120} w'(t)dt$ represent?

Finding the Displacement of a Particle Given the Velocity

620–624 *A particle moves according to the given velocity function over the given interval. Find the displacement of the particle.* ***Remember:*** *Displacement is the change in position, and velocity is the rate of change in position with respect to time.*

620. $v(t) = 2t - 4,\ 0 \le t \le 5$

621. $v(t) = t^2 - t - 6,\ 2 \le t \le 4$

622. $v(t) = 2\cos t,\ 0 \le t \le \pi$

623. $v(t) = \sin t - \cos t,\ \frac{-\pi}{6} \le t \le \frac{\pi}{2}$

624. $v(t) = \sqrt{t} - 4,\ 1 \le t \le 25$

Finding the Distance Traveled by a Particle Given the Velocity

625–629 *A particle moves according to the given velocity function over the given interval. Find the total distance traveled by the particle.* ***Remember:*** *Velocity is the rate of change in position with respect to time.*

625. $v(t) = 2t - 4,\ 0 \le t \le 5$

626. $v(t) = t^2 - t - 6,\ 2 \le t \le 4$

627. $v(t) = 2\cos t,\ 0 \le t \le \pi$

628. $v(t) = \sin t - \cos t,\ \frac{-\pi}{6} \le t \le \frac{\pi}{2}$

629. $v(t) = \sqrt{t} - 4,\ 1 \le t \le 25$

Finding the Displacement of a Particle Given Acceleration

630–632 *A particle moves according to the given acceleration function over the given interval. First, find the velocity function. Then find the displacement of the particle.* ***Remember:*** *Displacement is the change in position, velocity is the rate of change in position with respect to time, and acceleration is the rate of change in velocity with respect to time.*

630. $a(t) = t + 2,\ v(0) = -6,\ 0 \le t \le 8$

631. $a(t) = 2t + 1,\ v(0) = -12,\ 0 \le t \le 5$

632. $a(t) = \sin t + \cos t,\ v\left(\frac{\pi}{4}\right) = 0,\ \frac{\pi}{6} \le t \le \pi$

Finding the Distance Traveled by a Particle Given Acceleration

633–635 *A particle moves according to the given acceleration function over the given interval. Find the total distance traveled by the particle.* ***Remember:*** *Displacement is the change in position, velocity is the rate of change in position with respect to time, and acceleration is the rate of change in velocity with respect to time.*

633. $a(t) = t + 2,\ v(0) = -6,\ 0 \le t \le 8$

634. $a(t) = 2t + 1,\ v(0) = -12,\ 0 \le t \le 5$

635. $a(t) = \sin t + \cos t,\ v\left(\frac{\pi}{4}\right) = 0,\ \frac{\pi}{6} \le t \le \pi$

Chapter 11

Applications of Integration

This chapter presents questions related to applications of integrals: finding the area between curves, finding the volumes of a solid, and calculating the work done by a varying force. The work problems contain a variety of questions, all of which apply to a number of real-life situations and relate to questions that you may encounter in a physics class. At the end of the chapter, you answer questions related to finding the average value of a function on an interval.

The Problems You'll Work On

In this chapter, you see a variety of applications of the definite integral:

- Finding areas between curves
- Using the disk/washer method to find volumes of revolution
- Using the shell method to find volumes of revolution
- Finding volumes of solids using cross-sectional slices
- Finding the amount of work done when applying a force to an object
- Finding the average value of a continuous function on an interval

What to Watch Out For

Here are a few things to consider for the problems in this chapter:

- Make graphs for the area and volume problems to help you visualize as much as possible.
- Don't get the formulas and procedures for the disk/washer method mixed up with the shell method; it's easy to do! For example, when rotating regions about a horizontal line using disks/washers, your curve should be of the form $y = f(x)$, but if you're using shells, your curve should be of the form $x = g(y)$. When rotating a region about a vertical line and using disks/washers, your curve should be of the form $x = g(y)$, but if you're using shells, your curve should be of the form $y = f(x)$.
- Some of the volume of revolution problems can be solved using either the disk/washer method or the shell method; other problems can be solved easily only by using one method. Pay attention to which problems seem to be doable using either method and which ones do not.
- The work problems often give people a bit of a challenge, so don't worry if your first attempt isn't correct. Keep trying!

Areas between Curves

636–661 *Find the area of the region bounded by the given curves. (**Tip:** It's often useful to make a rough sketch of the region.)*

636. $y = x^2$, $y = x^4$

637. $y = x$, $y = \sqrt{x}$

638. $y = \cos x + 1$, $y = x$, $x = 0$, $x = 1$

639. $x = y^2 - y$, $x = 3y - y^2$

640. $x + 1 = y^2$, $x = \sqrt{y}$, $y = 0$, $y = 1$

641. $x = 1 + y^2$, $y = x - 7$

642. $x = y^2$, $x = 3y - 2$

643. $x = 2y^2$, $x + y = 1$

644. $y = 2x$, $y = 8 - x^2$

645. $x = 2 - y^2$, $x = y^2 - 2$

646. $y = 14 - x^2$, $y = x^2 - 4$

647. $x = y$, $4x + y^2 = -3$

648. $x = 1 + \sqrt{y}$, $x = \frac{3 + y}{3}$

649. $y = x - \frac{\pi}{2}$, $y = \cos x$, $x = 0$, $x = \pi$

650. $y = x^3 - x$, $y = 2x$

651. $x + y = 0$, $x = y^2 + 4y$

652. $x = \sqrt{y + 3}$, $x = \frac{y + 3}{2}$

653. $y = \sin x,\ y = \cos x,\ x = -\frac{\pi}{4},\ x = \frac{\pi}{2}$

654. $x = y^2,\ x = \sqrt{y},\ y = 0,\ y = 2$

655. $x = y^2 - y,\ x = 4y$

656. $y = x - 1,\ y^2 = 2x + 6$

657. $y = x,\ x + 2y = 0,\ 2x + y = 3$

658. $y = \sqrt{x+4},\ y = \frac{x+4}{2}$

659. $y = |2x|,\ y = x^2 - 3$

660. $y = \cos x,\ y = \sin 2x,\ x = 0,\ x = \frac{\pi}{2}$

661. $y = 2e^{2x},\ y = 3 - 5e^{x},\ x = 0$

Finding Volumes Using Disks and Washers

662–681 *Find the volume of the solid obtained by revolving the indicated region about the given line. (**Tip:** Making a rough sketch of the region that's being rotated is often useful.)*

662. The region is bounded by the curves $y = x^4$, $x = 1$, and $y = 0$ and is rotated about the x-axis.

663. The region is bounded by the curves $x = \sqrt{\sin y}$, $x = 0$, $y = 0$, and $y = \pi$ and is rotated about the y-axis.

664. The region is bounded by the curves $y = \frac{1}{x}$, $x = 3$, $x = 5$, and $y = 0$ and is rotated about the x-axis.

665. The region is bounded by the curves $y = \frac{1}{\sqrt{x}}$, $x = 1$, $x = 3$, and $y = 0$ and is rotated about the x-axis.

666. The region is bounded by the curves $y = \csc x$, $x = \frac{\pi}{4}$, $x = \frac{\pi}{2}$, and $y = 0$ and is rotated about the x-axis.

667. The region is bounded by the curves $x + 4y = 4$, $x = 0$, and $y = 0$ and is rotated about the x-axis.

668. The region is bounded by the curves $x = y^2 - y^3$ and $x = 0$ and is rotated about the y-axis.

669. The region is bounded by the curves $y = \sqrt{x-1}$, $y = 0$, and $x = 5$ and is rotated about the x-axis.

670. The region is bounded by the curves $y = 4 - \frac{x^2}{4}$ and $y = 2$ and is rotated about the x-axis.

671. The region is bounded by the curves $x = y^{2/3}$, $x = 0$, and $y = 8$ and is rotated about the y-axis.

672. The region is bounded by the curves $y = \sqrt{r^2 - x^2}$ and $y = 0$ and is rotated about the x-axis.

673. The region is bounded by the curves $y = \sin x$, $y = \cos x$, $x = 0$, and $x = \frac{\pi}{2}$ and is rotated about the x-axis.

674. The region is bounded by the curves $y = \frac{1}{1+x^2}$, $y = 0$, $x = 0$, and $x = 1$ and is rotated about the x-axis.

675. The region is bounded by the curves $y = 3 + 2x - x^2$ and $x + y = 3$ and is rotated about the x-axis.

676. The region is bounded by the curves $y = x^2$ and $x = y^2$ and is rotated about the y-axis.

677. The region is bounded by the curves $y = x^{2/3}$, $y = 1$, and $x = 0$ and is rotated about the line $y = 2$.

678. The region is bounded by the curves $y = x^{2/3}$, $y = 1$, and $x = 0$ and is rotated about the line $x = -1$.

679. The region is bounded by $y = \sec x$, $y = 0$, and $0 \le x \le \frac{\pi}{3}$ and is rotated about the line $y = 4$.

680. The region is bounded by the curves $x = y^2$ and $x = 4$ and is rotated about the line $x = 5$.

681. The region is bounded by the curves $y = e^{-x}$, $y = 0$, $x = 0$, and $x = 1$ and is rotated about the line $y = -1$.

Finding Volume Using Cross-Sectional Slices

682–687 *Find the volume of the indicated region using the method of cross-sectional slices.*

682. The base of a solid C is a circular disk that has a radius of 4 and is centered at the origin. Cross-sectional slices perpendicular to the x-axis are squares. Find the volume of the solid.

683. The base of a solid C is a circular disk that has a radius of 4 and is centered at the origin. Cross-sectional slices perpendicular to the x-axis are equilateral triangles. Find the volume of the solid.

684. The base of a solid S is an elliptical region with the boundary curve $4x^2 + 9y^2 = 36$. Cross-sectional slices perpendicular to the y-axis are squares. Find the volume of the solid.

685. The base of a solid S is triangular with vertices at (0, 0), (2, 0), and (0, 4). Cross-sectional slices perpendicular to the y-axis are isosceles triangles with height equal to the base. Find the volume of the solid.

686. The base of a solid S is an elliptical region with the boundary curve $4x^2 + 9y^2 = 36$. Cross-sectional slices perpendicular to the x-axis are isosceles right triangles with the hypotenuse as the base. Find the volume of the solid.

687. The base of a solid S is triangular with vertices at (0, 0), (2, 0), and (0, 4). Cross-sectional slices perpendicular to the y-axis are semicircles. Find the volume of the solid.

Finding Volumes Using Cylindrical Shells

688–711 *Find the volume of the region bounded by the given functions using cylindrical shells. Give an exact answer. (**Tip:** Making a rough sketch of the region that's being rotated is often useful.)*

688. The region is bounded by the curves $y = \frac{1}{x}$, $y = 0$, $x = 1$, and $x = 3$ and is rotated about the y-axis.

689. The region is bounded by the curves $y = x^2$, $y = 0$, and $x = 2$ and is rotated about the y-axis.

690. The region is bounded by the curves $x = \sqrt[3]{y}$, $x = 0$, and $y = 1$ and is rotated about the x-axis.

691. The region is bounded by the curves $y = x^2$, $y = 0$, and $x = 2$ and is rotated about the line $x = -1$.

692. The region is bounded by the curves $y = 2x$ and $y = x^2 - 4x$ and is rotated about the y-axis.

693. The region is bounded by the curves $y = x^4$, $y = 16$, and $x = 0$ and is rotated about the x-axis.

694. The region is bounded by the curves $x = 5y^2 - y^3$ and $x = 0$ and is rotated about the x-axis.

695. The region is bounded by the curves $y = x^2$ and $y = 4x - x^2$ and is rotated about the line $x = 4$.

696. The region is bounded by the curves $y = 1 + x + x^2$, $x = 0$, $x = 1$, and $y = 0$ and is rotated about the y-axis.

697. The region is bounded by the curves $y = 4x - x^2$, $x = 0$, and $y = 4$ and is rotated about the y-axis.

698. The region is bounded by the curves $y = \sqrt{9 - x}$, $x = 0$, and $y = 0$ and is rotated about the x-axis.

699. The region is bounded by the curves $y = 1 - x^2$ and $y = 0$ and is rotated about the line $x = 2$.

700. The region is bounded by the curves $y = 5 + 3x - x^2$ and $2x + y = 5$ and is rotated about the y-axis.

701. The region is bounded by the curves $x + y = 5$, $y = x$, and $y = 0$ and is rotated about the line $x = -1$.

702. The region is bounded by the curves $y = \sin(x^2)$, $x = 0$, $x = \sqrt{\pi}$, and $y = 0$ and is rotated about the y-axis.

703. The region is bounded by the curves $x = e^y$, $x = 0$, $y = 0$, and $y = 2$ and is rotated about the x-axis.

704. The region is bounded by the curves $y = e^{-x^2}$, $y = 0$, $x = 0$, and $x = 3$ and is rotated about the y-axis.

705. The region is bounded by the curves $x = y^3$ and $y = x^2$ and is rotated about the line $x = -1$.

706. The region is bounded by the curves $y = \frac{1}{x}$, $y = 0$, $x = 1$, and $x = 3$ and is rotated about the line $x = 4$.

707. The region is bounded by the curves $y = \sqrt{x}$ and $y = x^3$ and is rotated about the line $y = 1$.

708. The region is bounded by the curves $y = \sqrt{x+2}$, $y = x$, and $y = 0$ and is rotated about the x-axis.

709. The region is bounded by the curves $y = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$, $y = 0$, $x = 0$, and $x = 1$ and is rotated about the y-axis.

710. The region is bounded by the curves $y = \sqrt{x}$, $y = \ln x$, $x = 1$, and $x = 2$ and is rotated about the y-axis.

711. The region is bounded by the curves $x = \cos y$, $y = 0$, and $y = \frac{\pi}{2}$ and is rotated about the x-axis.

Work Problems

712–735 *Find the work required in each situation. Note that if the force applied is constant, work equals force times displacement ($W = Fd$); if the force is variable, you use the integral $W = \int_a^b f(x)dx$, where $f(x)$ is the force on the object at x and the object moves from $x = a$ to $x = b$.*

712. In joules, how much work do you need to lift a 50-kilogram weight 3 meters from the floor? (***Note:*** The acceleration due to gravity is 9.8 meters per second squared.)

713. In joules, how much work is done pushing a wagon a distance of 12 meters while exerting a constant force of 800 newtons in the direction of motion?

714. A heavy rope that is 30 feet long and weighs 0.75 pounds per foot hangs over the edge of a cliff. In foot-pounds, how much work is required to pull all the rope to the top of the cliff?

715. A heavy rope that is 30 feet long and weighs 0.75 pounds per foot hangs over the edge of a cliff. In foot-pounds, how much work is required to pull only half of the rope to the top of the cliff?

716. A heavy industrial cable weighing 4 pounds per foot is used to lift a 1,500-pound piece of metal up to the top of a building. In foot-pounds, how much work is required if the building is 300 feet tall?

717. A 300-pound uniform cable that's 150 feet long hangs vertically from the top of a tall building. How much work is required to lift the cable to the top of the building?

718. A container measuring 4 meters long, 2 meters wide, and 1 meter deep is full of water. In joules, how much work is required to pump the water out of the container? (***Note:*** The density of water is 1,000 kilograms per cubic meter, and the acceleration due to gravity is 9.8 meters per second squared.)

719. If the work required to stretch a spring 2 feet beyond its natural length is 14 foot-pounds, how much work is required to stretch the spring 18 inches beyond its natural length? (***Note:*** For a spring, force equals the spring constant k multiplied by the spring's displacement from its natural length: $F(x) = kx$.)

720. A force of 8 newtons stretches a spring 9 centimeters beyond its natural length. In joules, how much work is required to stretch the spring from 12 centimeters beyond its natural length to 22 centimeters beyond its natural length? Round the answer to the hundredths place. (***Note:*** For a spring, force equals the spring constant k multiplied by the spring's displacement from its natural length: $F(x) = kx$. Also note that 1 newton-meter = 1 joule.)

721. A particle is located at a distance x meters from the origin, and a force of $2\sin\left(\frac{\pi x}{6}\right)$ newtons acts on it. In joules, how much work is done moving the particle from $x = 1$ to $x = 2$? The force is directed along the x-axis. Find an exact answer. (***Note:*** 1 newton-meter = 1 joule.)

722. Five joules of work is required to stretch a spring from its natural length of 15 centimeters to a length of 25 centimeters. In joules, how much work is required to stretch the spring from a length of 30 centimeters to a length of 42 centimeters? (***Note:*** For a spring, force equals the spring constant k multiplied by the spring's displacement from its natural length: $F(x) = kx$. Also note that 1 newton-meter = 1 joule.)

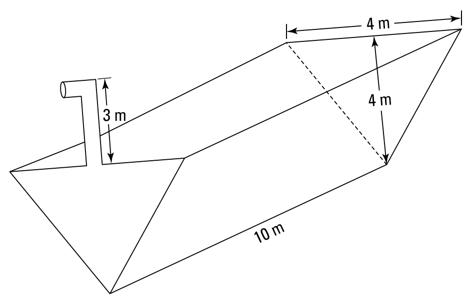
723. It takes a force of 15 pounds to stretch a spring 6 inches beyond its natural length. In foot-pounds, how much work is required to stretch the spring 8 inches beyond its natural length? (***Note:*** For a spring, force equals the spring constant k multiplied by the spring's displacement from its natural length: $F(x) = kx$.)

724. Suppose a spring has a natural length of 10 centimeters. If a force of 30 newtons is required to stretch the spring to a length of 15 centimeters, how much work (in joules) is required to stretch the spring from 15 centimeters to 20 centimeters? (***Note:*** For a spring, force equals the spring constant k multiplied by the spring's displacement from its natural length: $F(x) = kx$. Also note that 1 newton-meter = 1 joule.)

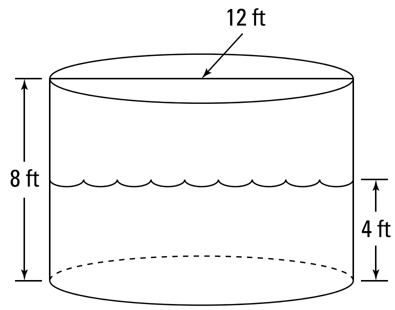
725. Suppose a 20-foot hanging chain weighs 4 pounds per foot. In foot-pounds, how much work is done in lifting the end of the chain to the top so that the chain is folded in half?

726. A 20-meter chain lying on the ground has a mass of 100 kilograms. In joules, how much work is required to raise one end of the chain to a height of 5 meters? Assume that the chain is L-shaped after being lifted with a remaining 15 meters of chain on the ground and that the chain slides without friction as its end is lifted. Also assume that the weight density of the chain is constant and is equal to $\left(\frac{100}{20}\text{ kg/m}\right)\left(9.8\text{ m/s}^2\right) = 49$ N/m. Round to the nearest joule. (***Note:*** 1 newton-meter = 1 joule.)

727. A trough has a triangular face, and the width and height of the triangle each equal 4 meters. The trough is 10 meters long and has a 3-meter spout attached to the top of the tank. If the tank is full of water, how much work is required to empty it? Round to the nearest joule. (***Note:*** The acceleration due to gravity is 9.8 meters per second squared, and the density of water is 1,000 kilograms per cubic meter.)



728. A cylindrical storage container has a diameter of 12 feet and a height of 8 feet. The container is filled with water to a height of 4 feet. How much work is required to pump all the water out over the side of the tank? Round to the nearest foot-pound. (***Note:*** Water weighs 62.5 pounds per cubic foot.)



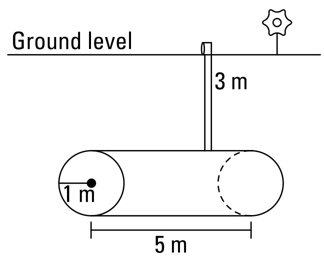
729. A thirsty farmer is using a rope of negligible weight to pull up a bucket that weighs 5 pounds from a well that is 100 feet deep. The bucket is filled with 50 pounds of water, but as the unlucky farmer pulls up the bucket at a rate of 2 feet per second, water leaks out at a constant rate and finishes draining just as the bucket reaches the top of the well. In foot-pounds, how much work has the thirsty farmer done?

730. Ten joules of work is needed to stretch a spring from 8 centimeters to 10 centimeters. If 14 joules of work is required to stretch the spring from 10 centimeters to 12 centimeters, what is the natural length of the spring in centimeters? (***Note:*** For a spring, force equals the spring constant k multiplied by the spring's displacement from its natural length: $F(x) = kx$. Also note that 1 newton-meter = 1 joule.)

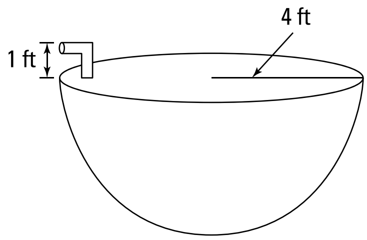
731. A cylindrical storage container has a diameter of 12 feet and a height of 8 feet. The container is filled with water to a distance of 4 feet from the top of the tank. Water is being pumped out, but the pump breaks after $13{,}500\pi$ foot-pounds of work has been completed. In feet, how far is the remaining water from the top of the tank? Round your answer to the hundredths place.

732. Twenty-five joules of work is needed to stretch a spring from 40 centimeters to 60 centimeters. If 40 joules of work is required to stretch the spring from 60 centimeters to 80 centimeters, what is the natural length of the spring in centimeters? Round the answer to two decimal places. (***Note:*** For a spring, force equals the spring constant k multiplied by the spring's displacement from its natural length: $F(x) = kx$. Also note that 1 newton-meter = 1 joule.)

733. A cylindrical storage tank with a radius of 1 meter and a length of 5 meters is lying on its side and is full of water. If the top of the tank is 3 meters below ground, how much work in joules will it take to pump all the water to ground level? (***Note:*** The acceleration due to gravity is 9.8 meters per second squared, and the density of water is 1,000 kilograms per cubic meter.)



734. A tank that has the shape of a hemisphere with a radius of 4 feet is full of water. If the opening to the tank is 1 foot above the top of the tank, how much work in foot-pounds is required to empty the tank?



735. An open tank full of water has the shape of a right circular cone. The tank is 10 feet across the top and 6 feet high. In foot-pounds, how much work is done in emptying the tank by pumping the water over the top edge? Round to the nearest foot-pound. (***Note:*** Water weighs 62.5 pounds per cubic foot.)

Average Value of a Function

736–741 *Find the average value of the function on the given interval by using the formula* $f_{avg} = \frac{1}{b-a}\int_a^b f(x)dx$.

736. $f(x) = x^3$, $[-1, 2]$

737. $f(x) = \sin x$, $\left[0, \frac{3\pi}{2}\right]$

738. $f(x) = (\sin^3 x)(\cos x)$, $\left[0, \frac{\pi}{2}\right]$

739. $g(x) = x^2\sqrt{1+x^3}$, $[0, 2]$

740. $y = \sinh x \cosh x$, $[0, \ln 3]$

741. $f(r) = \frac{5}{(1+r)^2}$, $[1, 4]$

742–747 *Solve the problem using the average value formula.*

742. The linear density of a metal rod measuring 8 meters in length is $f(x) = \frac{14}{\sqrt{x+2}}$ kilograms per meter, where x is measured in meters from one end of the rod. Find the average density of the rod.

743. Find all numbers d such that the average value of $f(x) = 2 + 4x - 3x^2$ on $[0, d]$ is equal to 3.

744. Find all numbers d such that the average value of $f(x) = 3 + 6x - 9x^2$ on $[0, d]$ is equal to -33.

745. Find all values of c in the given interval such that $f_{avg} = f(c)$ for the function $f(x) = \frac{4(x^2+1)}{x^2}$ on $[1, 3]$.

746. Find all values of c in the given interval such that $f_{avg} = f(c)$ for the function $f(x) = \frac{1}{\sqrt{x}}$ on $[4, 9]$.

747. Find all values of c in the given interval such that $f_{avg} = f(c)$ for the function $f(x) = 5 - 3x^2$ on $[-2, 2]$.

748–749 *Use the average value formula.*

748. For the function $f(x) = x \sin x$ on the interval $\left[0, \frac{\pi}{2}\right]$, find the average value.

749. For the function $y = \sqrt{x}$ on the interval $\left[0, \frac{\pi}{4}\right]$, find the average value.

Chapter 12

Inverse Trigonometric Functions, Hyperbolic Functions, and L'Hôpital's Rule

This chapter looks at the very important inverse trigonometric functions and the hyperbolic functions. For these functions, you see lots of examples related to finding derivatives and integration as well. Although you don't spend much time on the hyperbolic functions in most calculus courses, the inverse trigonometric functions come up again and again; the inverse tangent function is especially important when you tackle the partial fraction problems of Chapter 14. At the end of this chapter, you experience a blast from the past: limit problems!

The Problems You'll Work On

This chapter has a variety of limit, derivative, and integration problems. Here's what you work on:

- Finding derivatives and antiderivatives using inverse trigonometric functions
- Finding derivatives and antiderivatives using hyperbolic functions
- Using L'Hôpital's rule to evaluate limits

What to Watch Out For

Here are a few things to consider for the problems in this chapter:

- The derivative questions just involve new formulas; the power, product, quotient, and chain rules still apply.
- Know the definitions of the hyperbolic functions so that if you forget any formulas, you can easily derive them. They're simply defined in terms of the exponential function, e^x.
- Although L'Hôpital's rule is great for many limit problems, make sure you have an indeterminate form before you use it, or you can get some very incorrect solutions.

Finding Derivatives Involving Inverse Trigonometric Functions

750–762 *Find the derivative of the given function.*

750. $y = 2\sin^{-1}(x-1)$

751. $y = 3\cos^{-1}\left(x^4 + x\right)$

752. $y = \sqrt{\tan^{-1} x}$

753. $y = \sqrt{1-x^2}\,\sin^{-1} x$

754. $y = \tan^{-1}(\cos x)$

755. $y = e^{\sec^{-1} t}$

Note: The derivative formula for $\sec^{-1} t$ varies, depending on the definition used. For this problem, use the formula $\frac{d}{dt}\sec^{-1} t = \frac{1}{t\sqrt{t^2-1}}$.

756. $y = \csc^{-1} e^{2x}$

757. $y = e^{x\sec^{-1} x}$

Note: The derivative formula for $\sec^{-1} x$ varies, depending on the definition used. For this problem, use the formula $\frac{d}{dx}\sec^{-1} x = \frac{1}{x\sqrt{x^2-1}}$.

758. $y = 4\arccos\frac{x}{3}$

759. $y = x\sin^{-1} x + \sqrt{1-x^2}$

760. $y = \cot^{-1} x + \cot^{-1}\frac{1}{x}$

761. $y = \tan^{-1} x + \frac{x}{1+x^2}$

762. $y = \tan^{-1}\left(x - \sqrt{1+x^2}\right)$

Finding Antiderivatives by Using Inverse Trigonometric Functions

763–774 *Find the indefinite integral or evaluate the definite integral.*

763. $\int_0^1 \frac{2}{x^2+1}\,dx$

764. $\int_{1/2}^{\sqrt{3}/2} \frac{5}{\sqrt{1-x^2}}\,dx$

765. $\int \frac{dx}{\sqrt{1-9x^2}}$

766. $\int_0^{1/2} \frac{\sin^{-1} x}{\sqrt{1-x^2}}\,dx$

767. $\int_0^{\pi/2} \frac{\cos x}{1+\sin^2 x}\,dx$

768. $\int \frac{1}{x\left(1+(\ln x)^2\right)}\,dx$

769. $\int \frac{dx}{\sqrt{x}\,(1+x)}$

770. $\int \frac{1}{x\sqrt{x^2-25}}\,dx$

771. $\int \frac{1}{(x-1)\sqrt{x^2-2x}}\,dx$

772. $\int \frac{e^{3x}}{\sqrt{1-e^{6x}}}\,dx$

773. $\int \frac{x+4}{x^2+4}\,dx$

774. $\int_2^3 \frac{2x-3}{\sqrt{4x-x^2}}\,dx$

Evaluating Hyperbolic Functions Using Their Definitions

775–779 *Use the definition of the hyperbolic functions to find the values.*

775. sinh 0

776. cosh (ln 2)

777. $\coth(\ln 6)$

778. $\tanh 1$

779. $\operatorname{sech}\frac{1}{2}$

Finding Derivatives of Hyperbolic Functions

780–789 *Find the derivative of the given function.*

780. $y = \cosh^2 x$

781. $y = \sinh(x^2)$

782. $y = \frac{1}{6}\operatorname{csch}(2x)$

783. $y = \tanh(e^x)$

784. $y = \tanh(\sinh x)$

785. $y = e^{\sinh(5x)}$

786. $y = \operatorname{sech}^4(10x)$

787. $y = \tanh\left(\sqrt{1+t^4}\right)$

788. $y = x^3 \sinh(\ln x)$

789. $y = \ln\left(\tanh\left(\frac{x}{3}\right)\right)$

Finding Antiderivatives of Hyperbolic Functions

790–799 *Find the antiderivative.*

790. $\int \sinh(1-3x)\,dx$

791. $\int \cosh^2(x-3)\sinh(x-3)\,dx$

792. $\int \coth x \, dx$

793. $\int \operatorname{sech}^2(3x-2)\,dx$

794. $\int \frac{\operatorname{sech}^2 x}{2+\tanh x}\,dx$

795. $\int x\cosh(6x)\,dx$

796. $\int x\operatorname{csch}^2\left(\frac{x^2}{5}\right)dx$

797. $\int \frac{\operatorname{csch}\left(\frac{1}{x^2}\right)\coth\left(\frac{1}{x^2}\right)}{x^3}\,dx$

798. $\int_{\ln 2}^{\ln 3} \frac{\cosh x}{\cosh^2 x - 1}\,dx$

799. $\int_0^{\ln 2} \frac{\cosh x}{\sqrt{9-\sinh^2 x}}\,dx$

Evaluating Indeterminate Forms Using L'Hôpital's Rule

800–831 *If the limit is an indeterminate form, evaluate the limit using L'Hôpital's rule. Otherwise, find the limit using any other method.*

800. $\lim\limits_{x\to -1} \frac{x^3+1}{x+1}$

801. $\lim\limits_{x\to 1} \frac{x^4-1}{x^3-1}$

802. $\lim\limits_{x\to 2} \frac{x-2}{x^2+x-6}$

803. $\lim\limits_{x\to\left(\frac{\pi}{2}\right)^-} \frac{\cos x}{1-\sin x}$

804. $\lim\limits_{x\to 0} \frac{1-\cos x}{\tan x}$

805. $\lim\limits_{x\to\infty} \frac{\ln x}{x^2}$

806. $\lim\limits_{x\to 1} \frac{\ln x}{\cos\left(\frac{\pi}{2}x\right)}$

807. $\lim_{x\to 0}\frac{\tan^{-1}x}{x}$

808. $\lim_{x\to 2^+}\left(\frac{8}{x^2-4}-\frac{x}{x-2}\right)$

809. $\lim_{x\to 0}\frac{\sin(4x)}{2x}$

810. $\lim_{x\to 0^+}x\ln x$

811. $\lim_{x\to\infty}\frac{\ln x^4}{x^3}$

812. $\lim_{x\to 0}\frac{\sin^{-1}x}{x}$

813. $\lim_{x\to\infty}\frac{\tan^{-1}x-\frac{\pi}{4}}{x-1}$

814. $\lim_{x\to\infty}\frac{\tan^{-1}x-\frac{\pi}{2}}{\frac{1}{1+x^3}}$

815. $\lim_{x\to\frac{\pi}{2}^-}(\sec x-\tan x)$

816. $\lim_{x\to\infty}x\sin\left(\frac{1}{x}\right)$

817. $\lim_{x\to-\infty}x^3e^x$

818. $\lim_{x\to\infty}\left(xe^{1/x}-x\right)$

819. $\lim_{x\to\infty}\frac{e^{3x+1}}{x^2}$

820. $\lim_{x\to 1}\frac{1-x+\ln x}{1+\cos\pi x}$

821. $\lim_{x\to 0}(1-5x)^{1/x}$

822. $\lim_{x\to\infty}x^{1/x^2}$

823. $\lim_{x\to 0^+}(\cos x)^{2/x}$

824. $\lim_{x\to\infty}\left(\frac{3x-1}{3x+4}\right)^{x-1}$

825. $\lim_{x\to 0^+}(2x)^{x^2}$

826. $\lim_{x\to 0^+}(\tan 3x)^x$

827. $\lim_{x\to 0^+}\tan x\ln x$

828. $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x$

829. $\lim_{x\to 0^+}\left(e^x+x\right)^{2/x}$

830. $\lim_{x\to 1^+}\left(\frac{x^2}{x^2-1}-\frac{1}{\ln x}\right)$

831. $\lim_{x\to 1^+}\left(\frac{1}{\ln x}-\frac{1}{x-1}\right)$

Chapter 13

U-Substitution and Integration by Parts

In this chapter, you encounter some of the more advanced integration techniques: *u*-substitution and integration by parts. You use *u*-substitution very, very often in integration problems. For many integration problems, consider starting with a *u*-substitution if you don't immediately know the antiderivative. Another common technique is integration by parts, which comes from the product rule for derivatives. One of the difficult things about these problems is that even when you know which procedure to use, you still have some freedom in how to proceed; what to do isn't always clear, so dive in and try different things.

The Problems You'll Work On

This chapter is the start of more challenging integration problems. You work on the following skills:

- Using *u*-substitution to find definite and indefinite integrals
- Using integration by parts to find definite and indefinite integrals

What to Watch Out For

Here are a few things to keep in mind while working on the problems in this chapter:

- Even if you know you should use a substitution, there may be different substitutions to try. As a rule, start simple and make your substitution more complex if your first choice doesn't work.
- When using a *u*-substitution, don't forget to calculate *du,* the differential.
- You can algebraically manipulate both *du* and the original *u*-substitution, so play with both!
- For the integration by parts problems, if your pick of *u* and *dv* don't seem to be working, try switching them.

Using u-Substitutions

832–857 *Use substitution to evaluate the integral.*

832. $\int \sin(5x)\,dx$

833. $\int (x+4)^{100}\,dx$

834. $\int 3x^2\sqrt{x^3+1}\,dx$

835. $\int \frac{\sec^2\sqrt{x}}{\sqrt{x}}\,dx$

836. $\int \frac{6}{(4+5x)^8}\,dx$

837. $\int \sin^7\theta\cos\theta\,d\theta$

838. $\int_{-\pi/3}^{\pi/3} \tan^3\theta\,d\theta$

839. $\int_1^{e^2} \frac{(\ln x)^3}{x}\,dx$

840. $\int \frac{dx}{4-3x}$

841. $\int \frac{\tan^{-1}x}{1+x^2}\,dx$

842. $\int \tan x\,dx$

843. $\int_0^{\pi/3} e^{\cos x}\sin x\,dx$

844. $\int_0^1 \sqrt{x}\cos\left(1+x^{3/2}\right)dx$

845. $\int \sqrt[7]{\tan x}\sec^2 x\,dx$

846. $\int \frac{\sin\left(\frac{\pi}{x}\right)}{x^2}\,dx$

847. $\int \frac{4+12x}{\sqrt{1+2x+3x^2}}\,dx$

848. $\int \sqrt[3]{\cot x}\csc^2 x\,dx$

849. $\int_0^{\sqrt{\pi}/2} x\sin\left(x^2\right)dx$

850. $\int \frac{3+x}{1+x^2}\,dx$

851. $\int_e^{e^2} \frac{dx}{x\sqrt{\ln x}}$

852. $\int \tan\theta\ln(\cos\theta)\,d\theta$

853. $\int x^2e^{-x^3}\,dx$

854. $\int_0^3 x\sqrt{x+1}\,dx$

855. $\int_0^1 x^5\sqrt[3]{x^3+1}\,dx$

856. $\int \frac{x}{\sqrt[6]{x+3}}\,dx$

857. $\int x^7\sqrt{x^4+1}\,dx$

Using Integration by Parts

858–883 *Use integration by parts to evaluate the integral.*

858. $\int x\cos(4x)\,dx$

859. $\int xe^x\,dx$

860. $\int x\sinh(2x)\,dx$

861. $\int_0^3 x6^x\,dx$

862. $\int x \sec x \tan x \, dx$

863. $\int x \sin(5x) \, dx$

864. $\int x \csc^2 x \, dx$

865. $\int x \sin x \, dx$

866. $\int x \csc x \cot x \, dx$

867. $\int \ln(3x+1) \, dx$

868. $\int \tan^{-1} x \, dx$

869. $\int_1^4 \frac{\ln x}{x^2} \, dx$

870. $\int_1^9 e^{\sqrt{x}} \, dx$

871. $\int_1^9 x^{3/2} \ln x \, dx$

872. $\int \cos^{-1} x \, dx$

873. $\int \sin^{-1}(5x) \, dx$

874. $\int_0^1 \frac{x^2}{e^{2x}} \, dx$

875. $\int_0^1 (x^2+1) e^{-x} \, dx$

876. $\int x^3 \cos(x^2) \, dx$

877. $\int x^2 \sin(mx) \, dx$, where $m \neq 0$

878. $\int_1^3 x^5 (\ln x)^2 dx$

879. $\int e^x \cos(2x)\, dx$

880. $\int \sin x \ln(\cos x)\, dx$

881. $\int \cos \sqrt{x}\, dx$

882. $\int x^4 (\ln x)^2\, dx$

883. $\int x \tan^{-1} x\, dx$

Chapter 14

Trigonometric Integrals, Trigonometric Substitution, and Partial Fractions

This chapter covers trigonometric integrals, trigonometric substitutions, and partial fractions — the remaining integration techniques you encounter in a second-semester calculus course (in addition to *u*-substitution and integration by parts; see Chapter 13). In a sense, these techniques are nothing fancy. For the trigonometric integrals, you typically use a *u*-substitution followed by a trigonometric identity, possibly throwing in a bit of algebra to solve the problem. For the trigonometric substitutions, you're often integrating a function involving a radical; by picking a clever substitution, you can often remove the radical and make the problem into a trigonometric integral and proceed from there. Last, the partial fractions technique simply decomposes a rational function into a bunch of simple fractions that are easier to integrate.

With that said, many of these problems have many steps and require you to know identities, polynomial long division, derivative formulas, and more. Many of these problems test your algebra and trigonometry skills as much as your calculus skills.

The Problems You'll Work On

This chapter finishes off the integration techniques that you see in a calculus class:

- Solving definite and indefinite integrals involving powers of trigonometric functions
- Solving definite and indefinite integrals using trigonometric substitutions
- Solving definite and indefinite integrals using partial fraction decompositions

What to Watch Out For

You can get tripped up in a lot of little places on these problems, but hopefully these tips will help:

- Not all of the trigonometric integrals fit into a nice mold. Try identities, *u*-substitutions, and simplifying the integral if you get stuck.
- You may have to use trigonometry and right triangles in the trigonometric substitution problems to recover the original variable.
- If you've forgotten how to do polynomial long division, you can find some examples in Chapter 1's algebra review.
- The trigonometric substitution problems turn into trigonometric integral problems, so make sure you can solve a variety of the latter problems!

Trigonometric Integrals

884–913 *Find the antiderivative or evaluate the definite integral.*

884. $\int_0^{3\pi/2} \sin^2(2\theta)\,d\theta$

885. $\int_0^{\pi/4} \cos^2\theta\,d\theta$

886. $\int \tan^2 x\,dx$

887. $\int \sec^4 t\,dt$

888. $\int \sin(3x)\sin(2x)\,dx$

889. $\int \cos(5x)\cos(2x)\,dx$

890. $\int \sin^4 x\cos x\,dx$

891. $\int \tan^2 x\sec^2 x\,dx$

892. $\int \sin(5x)\cos(4x)\,dx$

893. $\int \sec x \tan x \, dx$

894. $\int \sqrt{\csc^5 x} \csc x \cot x \, dx$

895. $\int \cos^3 x \sin^2 x \, dx$

896. $\int_0^{\pi/3} \tan^3 x \sec^4 x \, dx$

897. $\int \frac{\cot^3 x}{\sin^4 x} dx$

898. $\int_0^{\pi/2} \cos^3 x \, dx$

899. $\int \frac{1-\cos x}{\sin x} dx$

900. $\int (1+\sin\theta)^2 \, d\theta$

901. $\int \frac{\cos x - \sin x}{\sin(2x)} dx$

902. $\int \frac{\tan^3 x}{\cos^2 x} dx$

903. $\int x \sin^2 x \, dx$

904. $\int \sin^3 x \cos^3 x \, dx$

905. $\int \tan^3 x \sec x \, dx$

906. $\int \cot^3 x \csc x \, dx$

907. $\int \cos^3 x \sqrt{\sin x} \, dx$

908. $\int \cot^2 x \csc^4 x \, dx$

909. $\int \sin\theta \sin^5(\cos\theta)\,d\theta$

910. $\int \sin^5 x \cos^4 x\,dx$

911. $\int \sec^3 x\,dx$

912. $\int \frac{dx}{\sin x - 1}$

913. $\int \tan^3(4x)\sec^5(4x)\,dx$

Trigonometric Substitutions

914–939 *Evaluate the integral using a trigonometric substitution.*

914. $\int \frac{dx}{\sqrt{x^2+4}}$

915. $\int_0^{\sqrt{2}/2} \frac{x^2}{\sqrt{1-x^2}}\,dx$

916. $\int \frac{x}{\sqrt{x^2-1}}\,dx$

917. $\int \frac{1}{x^2\sqrt{1+x^2}}\,dx$

918. $\int \frac{1}{x^2\sqrt{x^2-16}}\,dx$

919. $\int \sqrt{9-x^2}\,dx$

920. $\int \sqrt{1-25x^2}\,dx$

921. $\int \frac{dx}{x\sqrt{7-x^2}}$

922. $\int \sqrt{1-5x^2}\,dx$

923. $\int \frac{dx}{x\sqrt{9-x^2}}$

924. $\int \frac{\sqrt{x^2-4}}{x^3}\,dx$

925. $\int x\sqrt{1-x^4}\,dx$

926. $\int_0^{\pi/2} \frac{\sin t}{\sqrt{1+\cos^2 t}}\,dt$

927. $\int \frac{\sqrt{x^2-1}}{x}\,dx$

928. $\int_1^2 \frac{1}{x^2\sqrt{x^2-1}}\,dx$

929. $\int \frac{x^2}{\left(4-x^2\right)^{3/2}}\,dx$

930. $\int \frac{1}{x^2\sqrt{4x^2-16}}\,dx$

931. $\int \frac{x^3}{\sqrt{x^2+16}}\,dx$

932. $\int \frac{x^5}{\sqrt{x^2+1}}\,dx$

933. $\int_0^3 x^3\sqrt{x^2+9}\,dx$

934. $\int x^2\sqrt{1-x^6}\,dx$

935. $\int \frac{dx}{\left(7-6x-x^2\right)^{3/2}}$

936. $\int \frac{dx}{x^2+4x+2}$

937. $\int_0^{3/2} x^3\sqrt{9-4x^2}\,dx$

938. $\int \sqrt{7+6x-x^2}\,dx$

939. $\int x^3\sqrt{16-x^2}\,dx$

Finding Partial Fraction Decompositions (without Coefficients)

940–944 *Find the partial fraction decomposition without finding the coefficients.*

940. $\frac{4x+1}{x^3(x+1)^2}$

941. $\frac{2x}{x^4-1}$

942. $\dfrac{5x^2+x-4}{(x+1)^2(x^2+5)^3}$

943. $\dfrac{4x^3+19}{x^2(x-1)^3(x^2+17)}$

944. $\dfrac{3x^2+4}{(x^2-9)(x^4+2x^2+1)}$

Finding Partial Fraction Decompositions (Including Coefficients)

945–949 *Find the partial fraction decomposition, including the coefficients.*

945. $\dfrac{1}{(x+2)(x-1)}$

946. $\dfrac{x+2}{x^3+x}$

947. $\dfrac{5x+1}{x^2-6x+9}$

948. $\dfrac{x^2+2}{x^4+4x^2+3}$

949. $\dfrac{x^2+1}{(x^2+5)^2}$

Integrals Involving Partial Fractions

950–958 *Evaluate the integral using partial fractions.*

950. $\int \dfrac{x}{x-5}\,dx$

951. $\int \dfrac{x^2}{x+6}\,dx$

952. $\int \dfrac{x-3}{(x+4)(x-5)}\,dx$

953. $\int_4^5 \dfrac{1}{x^2-1}\,dx$

954. $\int \dfrac{3x+5}{x^2+2x+1}\,dx$

955. $\int \dfrac{x^3-6x+5}{x^2-x-6}\,dx$

956. $\int \dfrac{8}{x^3+x}\,dx$

957. $\int \frac{6x^2+1}{x^4+6x^2+9}\,dx$

958. $\int \frac{1}{x^3+1}\,dx$

Rationalizing Substitutions

959–963 *Use a rationalizing substitution and partial fractions to evaluate the integral.*

959. $\int \frac{1}{x\sqrt{x+1}}\,dx$

960. $\int_4^9 \frac{\sqrt{x}}{x-1}\,dx$

961. $\int \frac{2}{\sqrt{x}-\sqrt[3]{x}}\,dx$

962. $\int \frac{1}{\sqrt[3]{x}-\sqrt[4]{x}}\,dx$

963. $\int \frac{e^{2x}}{e^{2x}+3e^x+2}\,dx$

Chapter 15

Improper Integrals and More Approximating Techniques

The problems in this chapter involve improper integrals and two techniques to approximate definite integrals: Simpson's rule and the trapezoid rule. *Improper integrals* are definite integrals with limits thrown in, so those problems require you to make use of many different calculus techniques; they can be quite challenging. The last few problems of the chapter involve using Simpson's rule and the trapezoid rule to approximate definite integrals. When you know the formulas for these approximating techniques, the problems are more of an arithmetic chore than anything else.

The Problems You'll Work On

This chapter involves the following tasks:

- Solving improper integrals using definite integrals and limits
- Using comparison to show whether an improper integral converges or diverges
- Approximating definite integrals using Simpson's rule and the trapezoid rule

What to Watch Out For

Here are a few pointers to help you finish the problems in this chapter:

- Improper integrals involve it all: limits, l'Hôpital's rule, and any of the integration techniques.
- The formulas for Simpson's rule and the trapezoid rule are similar, so don't mix them up!
- If you're careful with the arithmetic on Simpson's rule and the trapezoid rule, you should be in good shape.

Convergent and Divergent Improper Integrals

964–987 *Determine whether the integral is convergent or divergent. If the integral is convergent, give the value.*

964. $\int_1^\infty \frac{1}{(x+1)^2}\,dx$

965. $\int_0^5 \frac{1}{x\sqrt{x}}\,dx$

966. $\int_0^2 \frac{1}{x}\,dx$

967. $\int_{-\infty}^1 e^{-4x}\,dx$

968. $\int_1^{17} (x-1)^{-1/4}\,dx$

969. $\int_{-\infty}^{-3} \frac{1}{x+1}\,dx$

970. $\int_{-\infty}^{2} (x^2-5)\,dx$

971. $\int_{-\infty}^{\infty} (3-x^4)\,dx$

972. $\int_2^\infty e^{-x/3}\,dx$

973. $\int_e^\infty \frac{1}{x(\ln x)^2}\,dx$

974. $\int_{-\infty}^{\infty} \frac{x^2}{1+x^6}\,dx$

975. $\int_0^\infty xe^{-2x}\,dx$

976. $\int_{-\infty}^{\infty} x^4 e^{-x^5}\,dx$

977. $\int_{-\infty}^{-4} \frac{3x}{1+x^4}\,dx$

978. $\int_2^\infty \frac{\ln x}{x^4}\,dx$

979. $\int_0^{\infty} \frac{x}{x^2+4}\,dx$

980. $\int_1^{8} \frac{1}{\sqrt[3]{x-8}}\,dx$

981. $\int_0^{\pi/2} \tan^2 x\,dx$

982. $\int_0^{\infty} \sin^2 x\,dx$

983. $\int_{-1}^{1} \frac{e^x}{e^x-1}\,dx$

984. $\int_4^{\infty} \frac{1}{x^2+x-6}\,dx$

985. $\int_0^{2} \frac{dx}{\sqrt{4-x^2}}$

986. $\int_{-\infty}^{\infty} e^{-|x|}\,dx$

987. $\int_3^{5} \frac{x}{\sqrt{x-3}}\,dx$

The Comparison Test for Integrals

988–993 *Determine whether the improper integral converges or diverges using the comparison theorem for integrals.*

988. $\int_1^{\infty} \frac{\sin^2 x}{1+x^2}\,dx$

989. $\int_1^{\infty} \frac{dx}{x^4+e^{3x}}$

990. $\int_1^{\infty} \frac{x+2}{\sqrt{x^4-1}}\,dx$

991. $\int_1^{\infty} \frac{\tan^{-1} x}{x^5}\,dx$

992. $\int_2^{\infty} \frac{x^2}{\sqrt{x^6-1}}\,dx$

993. $\int_1^{\infty} \frac{5+e^{-x}}{x}\,dx$

The Trapezoid Rule

994–997 *Use the trapezoid rule with the specified value of n to approximate the integral. Round to the nearest thousandth.*

994. $\int_0^6 \sqrt[3]{1+x^3}\,dx$ with $n = 6$

995. $\int_1^2 \frac{\ln x}{1+x^2}\,dx$ with $n = 4$

996. $\int_1^3 \frac{\cos x}{x}\,dx$ with $n = 8$

997. $\int_0^{1/2} \sin\sqrt{x}\,dx$ with $n = 4$

Simpson's Rule

998–1,001 *Use Simpson's rule with the specified value of n to approximate the integral. Round to the nearest thousandth.*

998. $\int_0^6 \sqrt[3]{1+x^3}\,dx$ with $n = 6$

999. $\int_1^2 \frac{\ln x}{1+x^2}\,dx$ with $n = 4$

1,000. $\int_1^3 \frac{\cos x}{x}\,dx$ with $n = 8$

1,001. $\int_0^{1/2} \sin\sqrt{x}\,dx$ with $n = 4$